Math 221 Test 1 Fall 1999

Name: _____ ID #: _____

Part I: Hand Calculations: NO CALCULATORS

1. [15 points] Fill in the blanks so that the equation below is true.

$$\begin{bmatrix} \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ 3w \\ x+y-2z \\ x+w+z+2y \end{bmatrix}$$

2. [15 points] Find the inverse of $\begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{bmatrix}$.

3. [15 points] Solve the system of equations below, expressing your answer in vector form. What is the rank of the matrix? What is the dimension of the solution set?

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 1 & 1 & 4 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$$

4. [15 points] Find an equation for the line formed from the intersection of the planes given by $2x + 3y + 4z = 7$ and $x - y + 2x = 0$. Express your answer in parametric form, *i.e.* $x(t) = ...$, $y(t) = ...$, $z(t) = ...$, where t is a parameter.

Part II: Theory: NO CALCULATORS

5. [5 points] What is the formal definition of matrix multiplication?

6. [5 points] Let r be a real number and let A and B be $m \times n$ matrices. Prove that $r(A + B) = rA + rB$. Give reasons for each step.

7. [10 points] Let A and B be $n \times n$ matrices. Suppose that the product AB is **not** invertible. Prove that either A or B is not invertible. Give reasons for each step.

Part III: Use Maple

8. [10 points] Find the equation of the polynomial of least degree which passes through the points (1,3), (2,5), (3,1), (5,-1). Plot its graph.

9. [10 points] Find all values of x for which the matrix below is not invertible.

$$\begin{bmatrix} 1 & x & 2 & 0 \\ 0 & 0 & 0 & 3-x \\ 1 & 1 & 4 & 0 \\ x & x & 0 & 1 \end{bmatrix}$$

Print your results. Be sure your name is displayed!