

Print Name: \_\_\_\_\_ Id number: \_\_\_\_\_

**NO CALCULATORS**

1. [5 points] Find the inverse of  $f(x) = 2x - 3$ .
2. [5 points] Graph two full cycles of  $y = 3 \sin 2x$ . Label the zeros.
3. [5 points] What is the formal definition of matrix multiplication. Use summation notation.
4. [5 points] Prove, or disprove, the claim that

$$[C^{-1}(A + B)^T]^{-1} = (A^T + B^T)^{-1} C.$$

(Assume the various operations are well defined for the matrices  $A$ ,  $B$  and  $C$ .) You must give a written reason for each step.

5. [10 points] Prove this statement: If  $A$  and  $B$  are nonsingular matrices, then  $AB$  is nonsingular and  $(AB)^{-1} = B^{-1}A^{-1}$ . (This is Theorem 1.9(b) in your text.)

6. [15 points] Find the solution set for the matrix equation below. Express your answer in vector form, as done in class. State which variables, if any, are free. Circle the pivots in the reduced matrix.

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & 2 & 2 & -1 \\ 2 & 4 & 2 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

7. [10 points] Let  $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$ . Find the inverse of  $A$  or show that  $A$  is not invertible. Label all steps.

8. [15 points] Find a parametric equation for the line in  $\mathbb{R}^3$  that passes through the two points  $(2, 1, 0)$  and  $(1, 3, 1)$ . Determine whether or not this line also passes through the origin.

9. [10 points] Find the determinates of the two matrices below. Hint: Think before you jump.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -7 \\ 0 & 3 & \pi \\ 0 & 0 & 4 \end{bmatrix}$$

10. [20 points] A manufacturer make three different types of chemical products: A, B and C. Each product must go through two processing machines: X and Y. The products require the following times in machines X and Y:
- (a) One ton of A requires 2 hours in machine X and 2 hours in machine Y.
  - (b) One ton of B requires 3 hours in machine X and 2 hours in machine Y.
  - (c) One ton of C requires 4 hours in machine X and 3 hours in machine Y.

Machine X is available 80 hours per week and machine Y is available 60 hours per week. Since management does not want to keep the expensive machines X and Y idle, it would like to know how many tons of each product to make so that the machines are fully utilized. It is assumed that the manufacturer can sell as much of the product as is made. Be sure to explain what your answer means in terms management can understand.