

Name: \_\_\_\_\_ ID #: \_\_\_\_\_

**Scientific calculators only. No graphing or programmable calculators.**

(1) [5 points] Compute the determinant of  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 2 & 4 & 6 & 9 \end{bmatrix}$ .

(2) [10 points] Find the eigenvalues of  $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -4 & -17 & 8 \end{bmatrix}$ . Hint: one of them is  $\lambda = 4$ .

- (3) [15 points] The matrix below has eigenvalues 2 and 3. Find a basis for the eigenspace of each.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 4 & 6 & 3 \end{bmatrix}$$

(4) [5 points] Find the matrix for  $R_{\frac{\pi}{4}}$ , the linear transformation that rotates vectors in the plane by  $45^\circ$  counter clockwise.

(5) [5+5+10 points]

(a) Let  $T(x, y, z) = (3x - 2z, x - z + 3y, 0, 7z + x)$ . Find the matrix that represents  $T$ .

(b) Let  $T(x, y) = (x + y + 1, y)$ . Show by example that  $T$  is not linear.

(c) Let  $T(x, y) = (x + 2y, 2x + y)$ . Show that  $T$  is one-to-one and onto. Find  $T^{-1}$ .

(6) [15 points; 5 each] Clearly justify your answers.

(a) Is  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 7 \end{bmatrix} \right\}$  a basis for  $\mathbb{R}^3$ ?

(b) Is  $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$  a basis for  $\mathbb{R}^2$ ?

(c) Find a basis for the vector subspace of  $\mathbb{R}^4$  spanned by

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix} \right\}.$$

- (7) [15 points] Find bases for the row, column and null spaces of the matrix  $A$  below; Below it is  $\text{rref}(A)$ .

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 3 & 1 \\ 2 & 4 & 6 & 2 & 2 & 6 & 6 \\ 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 3 & 6 & 7 & 3 & 3 & 7 & 7 \\ 0 & 0 & 2 & 2 & 0 & 0 & 2 \\ 1 & 2 & 3 & 1 & 2 & 3 & 5 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(8) [5 points] Let  $S$  be a nonempty subset of  $\mathbb{R}^n$ . Prove that  $S^\perp$  is a vector space.

(9) [10 points] Prove: If  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation, then  $T$  is one-to-one if and only if  $\ker(T) = \{\mathbf{0}\}$ .