Theoretical Problems
Updated June 17 (#3 was changed.)
Part of Set #6

1. Let A be an $m \times n$ matrix, and B be a $n \times p$ matrix, so that the product AB is defined. Prove that if A has a row of zeros then so does AB.

2. Let $A = [a_{ij}]$ be a 2×3, $B = [b_{ij}]$ be a 3×4 matrix, and $C = [c_{ij}]$ be a 4×3 matrix. Prove that $A(BC) = (AB)C$.

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Use row operations to show that (i) if A has an inverse then it is given by,

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

and (ii) A is nonsingular if and only if $ad - bc \neq 0$.

Note, the quantity $ad - bc$ is called the *determinant* of A.