

Theoretical Problems

Set #11

1. Prove that if W is a subspace of a finite dimensional vector space V , and $\dim W = \dim V$, then $W = V$.
2. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and let $T = \{\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_2 + \mathbf{v}_1, \mathbf{v}_1\}$. Prove that S is linearly independent if and only if T is linearly independent.
3. Prove that for any $m \times n$ matrix A such that AA^T is nonsingular (*i.e.* invertible), then $\text{rank}(A) = m$.