

Theoretical Problems
Part of Set #18

1. Look up the definition of the **cross product** of two vectors in \mathbb{R}^3 (use the internet, your calculus book or a physics textbook). Let $\mathbf{v} = \langle a, b, c \rangle$ be a fixed vector in \mathbb{R}^3 and regard $\mathbf{x} = \langle x, y, z \rangle$ as variable vector in \mathbb{R}^3 . Let $T(\mathbf{x}) = \mathbf{v} \times \mathbf{x}$ (\mathbf{v} cross \mathbf{x}) define transformation from \mathbb{R}^3 to \mathbb{R}^3 . Either prove that T is linear or give an example demonstrating that it is not linear. If T is linear find its matrix representation with respect to the standard basis on \mathbb{R}^3 (now regarding \mathbf{x} as a column vector).
2. Let B be a nonstandard basis for \mathbb{R}^n . Let $\{\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_k\}$ be some other set of linearly independent vectors in \mathbb{R}^n expressed with respect to the standard basis (clearly $k \leq n$). Let $\mathbf{v}_i = [\mathbf{w}_i]_B$ for $i = 1, \dots, k$. Prove that $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly independent in \mathbb{R}^n . Hint: First do this for some specific value of n , say $n = 4$, then generalize.