Theoretical Problems
Part of Set #20

1. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a set of vectors in the vector space $\mathbb{R}^n$. Suppose $\mathbf{u} \in \mathbb{R}^n$ and is orthogonal (perpendicular) to each vector in S. Prove that $\mathbf{u}$ is orthogonal to each vector in span (S).

2. Let A be a square nonsingular matrix and suppose that $A^2 = A$. What is $\det(A)$? Be sure to prove your claim. Give an example of a 3×3 matrix with this property besides the identity matrix.

Errata. The second problem should have read: Let A be a square matrix and suppose that $A^2 = A$. What can you say about $\det(A)$? [It is 0 or 1.] Be sure to prove your claim.
Give an example of a 3×3 matrix A such that $A^2 = A$ besides the identity matrix or the zero matrix.