

Theoretical Problems
Part of Set #23

1. Let B and C be 2×2 matrices, and let O be the 2×2 matrix with all 0 entries. Let A be the 4×4 matrix given by

$$A = \begin{bmatrix} B & O \\ O & C \end{bmatrix}.$$

Prove that $\det(A) = \det(B)\det(C)$.

2. Let A be an $n \times n$ matrix. Suppose that A is **skew symmetric** (this means $A^T = -A$). Prove that if n is odd, then $\det(A) = 0$. Give an example of a 4×4 skew symmetric matrix that has determinant 12.
3. Let A , B and C be square matrices of the same size. Let $A \sim B$ mean A is similar to B . Prove each of the following statements.
- (a) A is similar to itself.
 - (b) If $A \sim B$ then $B \sim A$.
 - (c) If $A \sim B$ and $B \sim C$ then $A \sim C$.

(This means that similarity gives an **equivalence relation** on the set of $n \times n$ matrices for any n .)