

## Application: Curve Fitting

**Example 1.** Find a fourth degree polynomial,  $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ , which passes through the these points:  $(1, 2)$ ,  $(3, 8)$ ,  $(4, 1)$ ,  $(-1, 3)$ , and  $(-2, -1)$ .

*Solution.* The substitutions,  $f(1) = 2$ ,  $f(3) = 8, \dots, f(-2) = -1$ , yield the following equations,

$$\begin{array}{rccccrcr} a & + & b & + & c & + & d & + & e & = & 2 \\ 81a & + & 27b & + & 9c & + & 3d & + & e & = & 8 \\ 256a & + & 64b & + & 16c & + & 4d & + & e & = & 1 \\ a & - & b & + & c & - & d & + & e & = & 3 \\ 16a & - & 8b & + & 4c & - & 2d & + & e & = & -1 \end{array}$$

We set up the augmented matrix.

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 81 & 27 & 9 & 3 & 1 & 8 \\ 256 & 64 & 16 & 4 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 3 \\ 16 & -8 & 4 & -2 & 1 & -1 \end{array} \right]$$

Using a computer we find that there is a unique solution.  $a = -2411/11640$ ,  $b = 59/97$ ,  $c = 2611/2328$ ,  $d = -215/194$ , and  $e = 769/485$ .  $\square$

**Problem 1.** Find the unique quadratic polynomial that passes through the points  $(2,25)$ ,  $(5,16)$  and  $(6,3)$ .

**Problem 2.** Find the unique cubic polynomial that passes through the points  $(-2, -34)$ ,  $(-1, -8)$ ,  $(1,2)$  and  $(2,22)$ .

**Problem 3.** Find all third degree polynomials such that  $f(1) = 1$  and  $f(2) = -1$ .

**Problem 4.** Find all fourth degree polynomials such that  $f(-1) = f(1) = 0$ ,  $f'(0) = f''(0) = 0$ .

**Example 2.** Find the equation in standard form for the unique circle that passes through the three points  $(1, 1)$ ,  $(-2, 4)$  and  $(5, 6)$ .

*Solution.* We wish to find the equation for our circle in the form  $(x - h)^2 + (y - k)^2 = r^2$ . However, first consider the equation expanded form

$$x^2 + y^2 + ax + by + c = 0.$$

Substituting in the three given points yields three equations in the three unknowns.

$$\begin{aligned} a + b + c &= -2 \\ -2a + 4b + c &= -20 \\ 5a + 6b + c &= -61 \end{aligned}$$

The solution is  $a = -77$ ,  $b = 249/5$ , and  $c = 126/5$ . (Check this with a computer.) It is an algebra exercise to solve for  $h$ ,  $k$  and  $r$ . You should get,  $h = 38.5$ ,  $k = -24.9$ , and  $r = \sqrt{2039.26} \approx 45.158$ .  $\square$

**Problem 5.** Find the equation in standard form for the unique sphere that passes through the four points  $(1, 1, 1)$ ,  $(-2, 1, 4)$ ,  $(3, 2, 1)$ ,  $(4, 4, 5)$ .

**Project 1.** Study how to write procedures (programs) in Maple. Write a Maple procedure whose input is three points in the plane that outputs the equation in standard form for the unique circle passing through those points, or indicates no such circle exists. For “bonus” points, have Maple plot the circle, marking the three user input points.