Math 250 Quiz 4 Spring 2000

Name: ______________________ ID #: ______________________

NO CALCULATORS

1. [15 points] [Review] Find the arc length of the arc of the parabola $y = x^2$ between the between the points $(0,0)$ and $(1,1)$. Hint: You will need to use $\int \sec^3\theta\, d\theta = (\sec(\theta)\tan(\theta) + \ln(|\sec(\theta)+\tan(\theta)|))/2 + C$. If you can derive this you will get 5 points extra credit.

2. [20 points] Evaluate each series or show that it diverges.

   a) $\displaystyle\sum_{n=0}^{\infty}\left[\left(\frac{3}{4}\right)^n - 2\left(\frac{5}{6}\right)^n\right]$

   b) $\displaystyle\sum_{n=2}^{\infty}\frac{2}{n^2-1}$

3. [15 points] [Theory] Let a be any constant and suppose that $|r| < 1$. Prove that the geometric series $\sum_{n=0}^{\infty} ar^n$ converges and that its value is $\frac{a}{1-r}$.

b) Now suppose $|r| > 1$. What happens to the series and why?

c) What happens to the series when $r = \pm 1$. Explain why.

4. [30 points] For each series determine whether it converges or diverges. Justify your answer.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

b) $\displaystyle\sum_{n=2}^{\infty} \left[\frac{\ln(1+n^2)}{n}\right]^n$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3+3^n}$

5. [30 points] For each series determine whether it converges absolutely, converges conditionally, or diverges. Justify your answer.

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\ln(n^5)}$

b) $\sum_{n=0}^{\infty}(-1)^{n}\frac{n^2+1}{n^7+n^2+1}$

c) $\sum_{n=0}^{\infty}(-2)^{n+1}\frac{7}{\left(\sqrt{3}\right)^n}$