

Name: \_\_\_\_\_ ID #: \_\_\_\_\_

**CALCULATORS ALLOWED**

1. [20 points] Do the series below converge or diverge? Justify your answer.

a)  $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^4}$ . Hint:  $\ln n < n$ .

b)  $\sum_{n=1}^{\infty} \frac{n^2}{3^n + n}$ .

2. [20 points] Find the Taylor series of  $\arcsin x$ , centered about 0, expressing your answer in  $\Sigma$  notation. Hint: Use an integral.

Too Hard!

3. [20 points] Find the first three terms of the Taylor series of  $\ln(x)$ , centered about  $c = 2$ .

Too Easy!

4. [20 points] Recall that Taylor's remainder formula implies:

$$\text{Error of } P_n(x) \leq \text{Max of } \left| \frac{f^{(n+1)}(z)}{(n+1)!} (x-c)^{n+1} \right|$$

over  $z$  between  $x$  and  $c$ , where  $P_n(x)$  is the Taylor polynomial of  $f(x)$  centered about  $c$ .

- a) How many terms in the Taylor series of  $e^x$ , centered about  $c = 0$ , do you need to approximate  $e^{-2}$  to three decimal places?

- b) Estimate the error in the approximation,

$$\ln(0.8) = \ln(1 - 0.2) \approx -0.2 - \frac{1}{2}(0.2)^2 - \frac{1}{3}(0.2)^3.$$

Note: I am using the Taylor series of  $\ln(1+x)$  centered about 0.

5. [20 points] Consider  $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ . How many terms are needed to achieve an accuracy of .01 ? Hints: Use the integral test with remainder. You will need to use integration by parts. In the end you will not be able to solve for  $N$ , so use trail and error. The correct answer is in between 700 and 800 terms.

To learn and not think over what you have learned is perfectly useless. To think without having first learned is dangerous.  
— Confucius, as fictionalized in Gore Vidal's novel *Creation*, Book 6, Chapter 7.