Math 251
Practice Final Exam

1. [10 points] Let $\mathbf{u} = \langle 1,1,1\rangle$, $\mathbf{v} = \langle 0,2,2\rangle$, and $\mathbf{w} = \langle -1,0,3\rangle$.
(a) Find the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.
(b) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.
(c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$, in degrees to two decimal places.
Answers: 6, $\langle 0,1,1\rangle$, 47.87^o.

2. [20 points] An object moves according to $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$.
(a) Find the tangential and normal components of acceleration at $t = 1$. *Answers:* $\frac{4}{3}$, $\frac{2\sqrt{5}}{3}$.
(b) Find the unit tangent and unit normal vectors at $t = 1$. *Answers:* $\mathbf{T}(1) = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$, $\mathbf{N}(1) = \left\langle -\frac{4\sqrt{5}}{15}, \frac{\sqrt{5}}{3}, -\frac{2\sqrt{5}}{15}\right\rangle$.

3. [10 points] Let $\mathbf{r}(t) = \langle \cos\pi t, \sin\pi t, t\rangle$. Graph $\mathbf{r}(t)$ for $0 \le t \le 2$ and find the total length traced out. *Answer:* $2\sqrt{\pi^2+1}$.

4. [10 points] Find the minimum and maximum value of the function $f(x,y) = 4x^2 + 9y^2$ when x and y must satisfy $x^2 + y^2 = 1$. *Answer:* Max is 9. Min is 4.

5. [10 points] The sphere of radius 2 has density proportional to the distance to the z-axis. Find its mass and center of mass. *Answer:* Mass is $4k\pi^2$. Center of mass is (0,0,0).

6. [10 points] Find the area bounded by one loop of $r = 2\cos 2\theta$. *Answer:* $\pi/2$.

7. [10 points] Let $\mathbf{F} = k\mathbf{r}/||\mathbf{r}||^3$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find the work done by $\mathbf{F}$ in moving a particle along the straight line segment C from (0,4,0) to (0,4,3). *Answer:* $k/20$.

8. [10 points] Let $\mathbf{F} = \langle x, y, 3z\rangle$. Find the flux of $\mathbf{F}$ up through the proportion of the plane $2x + 3y + z = 12$ in the positive octant. *Answer:* 240.

9. [10 points] Find the volume bounded by the plane $z = y + 2$ and the paraboloid $z = x^2 + y^2$. *Answer:* $81\pi/32$.

10. [10 points] Let $\mathbf{r} = \langle x, y, z\rangle$. Prove that $\bigtriangledown \ln|\mathbf{r}| = \mathbf{r}/|\mathbf{r}|^2$.

11. [10 points] (a) Compute $\oint_C (x+y^2)\,dx + (y+x^2)\,dy$ where C is the boundary of the square $[-1,1]\times[-1,1]$ counterclockwise.
(b) Repeat where C is the boundary of the square $[0,1]\times[0,1]$ counterclockwise.
Answers: 0, 0.

12. [10 points] Let $\mathbf{F} = yz^3\mathbf{i} + (2z - x^4)\mathbf{j} + x\sin(y)\mathbf{k}$. Let S be the unit sphere centered at (0,0,0). Find the flux of $\mathbf{F}$ out through S. *Answer:* 0.

13. [10 points] Let $\mathbf{F} = \langle 3z, 5x, -2y \rangle$. Let C be the ellipse formed from the intersection of of the plane $z = y + 3$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above. Compute $\oint_C \mathbf{F} \cdot \mathbf{T} ds$. *Answer:* 2π.