

Math 251  
Practice Test 1a

1. [10 points] Prove that  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ , where  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are vectors in  $R^3$ .
2. [10 points] Prove that if  $\theta$  is the angle between two vectors  $\mathbf{u}$  and  $\mathbf{v}$ , then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}.$$

Hint: use the law of cosines.

3. [10 points] Let  $\mathbf{u} = \langle 2, -2, 1 \rangle$  and  $\mathbf{v} = \langle -3, 0, 3 \rangle$ .
  - a) Find the projection of  $\mathbf{v}$  in the direction of  $\mathbf{u}$ .
  - b) Find the area of the parallelogram spanned by  $\mathbf{v}$  and  $\mathbf{u}$ .
4. [10 points] Consider the points  $A(1, 0, 0)$ ,  $B(3, 0, -1)$ , and  $C(2, 1, -2)$ .
  - a) Find the area of the triangle with vertices  $A$ ,  $B$ , and  $C$ .
  - b) Find the equation in standard form of the plane that passes through the points  $A$ ,  $B$ , and  $C$ .
5. [10 points] Consider the surface given by  $x = y^2 + 4z^2$ .
  - a. Graph the slice for  $x = 1$ .
  - b. Graph the slice for  $y = 0$ .
  - c. Which type of surface is this?
6. [10 points] Let  $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + t \mathbf{k}$ . It determines a helix in  $R^3$ .
  - a. Compute  $\mathbf{T}(t)$ .
  - b. Find a set of parametric equations for the tangent line to this helix at the point corresponding to  $t = \pi/4$ .
7. [10 points] Find the curvature of  $\mathbf{r}(t) = e^t \cos t \mathbf{i} + e^t \sin t \mathbf{j} + e^t \mathbf{k}$ .
8. [10 points] Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2+y^2}$$

9. [10 points] Let  $f(x, y, z) = \cos(xy) \tan(xz^3)$ . Find  $f_{xy}$  and  $f_{zz}$ .
10. [10 points] Let  $f(x) = x^3$ .
  - a. Find the curvature  $\kappa(x)$  of  $f(x)$ .
  - b. Sketch the graph of  $y = \kappa(x)$ .