

Math 251
Practice Test 1b

- [10 points] Prove that $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \theta$, where θ is the angle between \mathbf{u} and \mathbf{v} .
- [10 points] Find the volume of the parallelepiped with adjacent sides given by the vectors $\langle 1, 0, 1 \rangle$, $\langle 1, 0, 0 \rangle$, and $\langle 1, 1, 1 \rangle$.
- [10 points] Find the distance between the point $Q(1, 5, -4)$ and the plane given by $3x - y + 2z = 6$. Hint: Pick some point A on the plane. Then project the vector \overrightarrow{QA} onto a normal vector for the plane.
- [10 points] Consider the surface given by $4x^2 - 3y^2 + 12z^2 + 12 = 0$.
 - Show that the slice for $y = 0$ is the empty set.
 - Which type of surface is it?
- [10 points] Consider the parametric equations $x(t) = 2 \sin^2 t$ and $y(t) = 2 \cos^2 t$. Sketch its graph in the xy -plane.
- [10 points] A particle of mass 1 is subjected to a force given by

$$\mathbf{F}(t) = \langle 1, t, e^t \rangle.$$

At time $t = 0$ the velocity vector is $\mathbf{0}$ (the zero vector). Set up an integral to find how far the particle travels in the next 10 seconds. **Do not solve it.**

- [10 points] Let $f(x, y, z) = \sin(yx) + \cos(xyz) + x^2y^3e^z$. Show directly that $f_{xz} = f_{zx}$.
- [10 points] Sketch the level curves of $z = e^{xy}$ for $z = 0, 1$, and 2 .
- [10 points] Find an equation for the line in R^3 going through the points $(1, -2, 3)$ and $(7, 2, 1)$.
- [10 points] Find a vector function that represents the curve formed by the intersection of the cylinder given by $x^2 + y^2 = 16$ and the plane given by $x + z = 5$.
- [11 points] Let $\mathbf{r}(t) = \langle 3t^2, -2t^2, t^2 \rangle$. Find the arc length from $t = 0$ to $t = 1$. (Answer is $\sqrt{14}$.)