Math 251

Practice Test 2b

1. [10 points] Determine if the limit $\lim_{(x,y)\to(0,0)} \frac{x^3 + x^2y}{x^2 + y^2}$ exists. If it does, find its value.

2. [10 points] Let $f(x, y, z) = x^3 + y^2z + 2$. Find the rate of change of the value of f in the direction $\mathbf{v} = \langle 1, 1, 1 \rangle$ at the point $(1, 2, 3)$.

3. [15 points] Let $z = f(x, y)$, $x = r\cos\theta$, and $y = r\sin\theta$.

   a) Use the Chain Rule to write $\partial z/\partial r$ and $\partial z/\partial\theta$.

   b) Verify that $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$.

4. [10 points] Find and classify any critical points of $f(x, y) = x^2y^2 + xy$.

5. [10 points] Sketch the region of integration and evaluate the double integral

$$\int_0^1 \int_y^{\sqrt{y}} x^2y^2 \, dx \, dy.$$

6. [15 points] Find the center of mass of the lamina corresponding to the parabolic region $0 \le y \le 4 - x^2$ where the density at the point (x, y) is proportional to the distance between (x, y) and the x-axis.

7. [15 points] Graph $r = 3\cos 2\theta$ in polar coordinates, for $0 \le \theta \le 2\pi$. Find the total area enclosed.

8. [15 points] Find the mass and center of mass of the lamina bounded by $y = 1/(1+x^2)$, $x = 1$ and $x = -1$, with density function $\rho = 3$.