## Grad Div Curl Worksheet

We define three scalar functions:

$$f(x,y,z) = x^3y + ye^{xz}, \quad g(x,y,z) = \sin xy + \cos y - z, \quad h(x,y,z) = x^2e^y \sin z.$$

We define three vector fields:

$$\mathbf{F}(x,y,z) = \left\langle y^2, xyz, xy^2e^z \right\rangle, \quad \mathbf{G}(x,y,z) = \left\langle \sin yz, e^{xy}, xyz \right\rangle, \quad \mathbf{H}(x,y,z) = \left\langle x^2, e^y, z^3 \right\rangle.$$

Do each of the problems below. If an operation is impossible explain why. Answers are on the next page. Don't peek until you are done.

1. Find a scalar function $p(x,y,z)$ such that $\mathbf{H} = \bigtriangledown p$.
2. Find a scalar function $p(x,y,z)$ such that $\mathbf{F} = \bigtriangledown p$.
3. $\bigtriangledown f$
4. $\bigtriangledown^2 f$
5. $\bigtriangledown \times \bigtriangledown f$
6. $\mathbf{grad}\, g$
7. $\bigtriangledown h$
8. $\mathbf{curl}\, \mathbf{F}$
9. $\bigtriangledown \cdot \mathbf{H}$
10. $\mathbf{div}\, h\mathbf{H}$
11. $\mathbf{curl}\, h$
12. $\mathbf{grad}\, h^2$
13. $\bigtriangledown \cdot (\mathbf{G} \times \mathbf{H})$
14. $\mathbf{curl}\, (\mathbf{F} + \mathbf{H})$
15. $\bigtriangledown \times (\bigtriangledown \cdot \mathbf{H})$
16. $\bigtriangledown (\mathbf{div}\, \mathbf{H})$
17. $\mathbf{div}\, (\bigtriangledown \cdot \mathbf{H})$
18. $\bigtriangledown^2 h + \mathbf{G}$
19. $\bigtriangledown^2 h + \mathbf{div}\, \mathbf{G}$
20. $\mathbf{curl}\, \mathbf{H} \times \mathbf{curl}\, \mathbf{G}$

*Answers*

1. $p(x, y, z) = \frac{1}{3}x^3 + e^y + \frac{1}{4}z^4$
2. Cannot be done since $\mathbf{curl}\,\mathbf{F} \neq \mathbf{0}$.
3. $\langle 3x^2y + yze^{xz}, x^3 + e^{xz}, xye^{xz} \rangle$
4. $6xy + (yz^2 + x^2y)e^{xz}$
5. $\mathbf{0}$
6. $\langle y\cos xy, x\cos xy - \sin y, -1 \rangle$
7. $\langle 2xe^y \sin z, x^2e^y \sin z, x^2e^y \cos z \rangle$
8. $\langle xy(2e^z - 1), -y^2e^z, y(z-2) \rangle$
9. $2x + e^y + 3z^2$
10. $(4x^3 + 2x^2e^y + 3x^2z^2)e^y \sin z + x^2z^3e^y \cos z$
11. Not defined since $h$ is not a vector field.
12. $\left\langle 4x^3e^{2y}\sin^2 z, 2x^4e^{2y}\sin z, 2x^4e^{2y}\sin z\cos z \right\rangle$
13. $yz^3e^{xy} - yze^y + x^3x - z^4\cos yz + ye^y\cos yz$
14. $\langle xy(2e^z - 1), -y^2e^z, y(z-2) \rangle$
15. Not defined since you cannot take the curl of a scalar function.
16. $\langle 2, e^y, 6z \rangle$
17. Not defined since you cannot take the divergence of a scalar function.
18. Not defined since you cannot add a scalar and a vector.
19. $2e^y \sin z + xe^{xy} + xy$
20. $\mathbf{0}$