Partial Derivatives Worksheet

1. Let $f(x, y, t) = x^3 t e^{y^2 t}$. Find f_x, f_t, f_y, f_{tt}, and f_{yt}.
2. Let $g(r, \theta, z) = r \sin(\theta z)$. Find g_z, g_{rr}, g_{zz}, g_θ, and $g_{\theta z}$.
3. Let $H(x, y, z) = \frac{xy}{x^2 z - y^2}$. Find H_x, H_z, and H_{zz}.
4. Let $\Omega(x, y, z) = xyz \sin\left(x^2 z e^{xy^2 z}\right)$. Find Ω_x and Ω_y.
5. Let $f(p, q, r) = \ln\left(pr^2 + q\cos(pqr)\right)$. Find f_p and f_q.
6. Let $A(k, p) = kp^2 \cos^3\left(k^5 e^{kp^2}\right)$. Find A_k and A_p.
7. Let $E(x, y) = x^2 y \tan(xy^2)$. Find E_x, E_y, E_{xy}, and E_{yy}.

Answers

1. $f_x = 3x^2te^{y^2t}$.
   $f_t = x^3e^{y^2t} + x^3y^2e^{y^2t}$.
   $f_y = 2x^3yte^{y^2t}$.
   $f_{tt} = x^3y^2e^{y^2t} + x^3y^2e^{y^2t} + x^3y^4te^{y^2t}$.
   $f_{yt} = 2x^3ye^{y^2t} + 2x^3y^3te^{y^2t}$.

2. $g_z = 2r\theta z\cos(\theta z^2)$.
   $g_{rr} = 0$.
   $g_{zz} = 2r\theta\cos(\theta z^2) - 4r\theta^2z^2\sin(\theta z^2)$.
   $g_\theta = rz^2\cos(\theta z^2)$.
   $g_{\theta z} = 2rz\cos(\theta z^2) - 2rz^3\theta\cos(\theta z^2)$.

3. $H_x = -\frac{x^2yz+y^3}{(x^2z-y^2)^2}$.
   $H_z = \frac{-x^2}{(x^2z-y^2)^2}$.
   $H_{zz} = \frac{x^4}{(x^2z-y^2)^4}$.

4. $\Omega_x = yz\sin\left(x^2ze^{xy^2z}\right) + \left[2x^2yz^2 + x^3y^3z^3\right]e^{xy^2z}\cos\left(x^2ze^{xy^2z}\right)$.
   $\Omega_y = xz\sin\left(x^2ze^{xy^2z}\right) + 2x^4y^2z^3e^{xy^2z}\cos\left(x^2ze^{xy^2z}\right)$.

5. $f_p = \frac{r^2-q^2r\sin(pqr)}{pr^2+q\cos(pqr)}$.
   $f_q = \frac{\cos(pqr)-pqr\sin(pqr)}{pr^2+q\cos(pqr)}$.

6. $A_k = p^2\cos^3\left(k^5e^{kp^2}\right) - \left[15k^5p^2 + 3p^4k^6\right]e^{kp^2}\cos^2\left(k^5e^{kp^2}\right)\sin\left(k^5e^{kp^2}\right)$.
   $A_p = k\cos^3\left(k^5e^{kp^2}\right) - 6k^7p^3e^{kp^2}\cos^2\left(k^5e^{kp^2}\right)\sin\left(k^5e^{kp^2}\right)$.

7. $E_x = 2xy\tan(xy^2) + x^2y^3\sec^2(xy^2)$.
   $E_y = x^2\tan(xy^2) + x^3y\sec^2(xy^2)$.
   $E_{xy} = 2x\tan(xy^2) + 7x^2y^2\sec^2(xy^2) + 4x^3y^4\sec^2(xy^2)\tan(xy^2)$.
   $E_{yy} = x^3(2y+1)\sec^2(xy^2) + 4x^4y^2\sec^2(xy^2)\tan(xy^2)$.