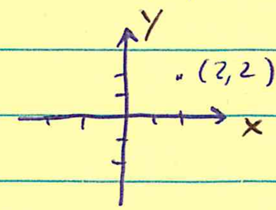
10.1 Rectangular 3-Dimension Coordinates

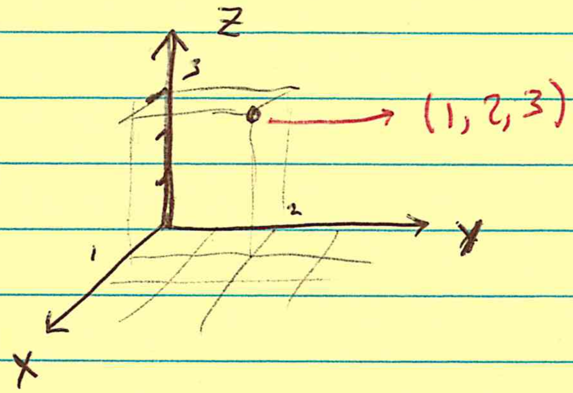
$\mathbb{R}$ = the real number line.

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$ = the xy-plane



$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x,y,z) \mid x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}\}$

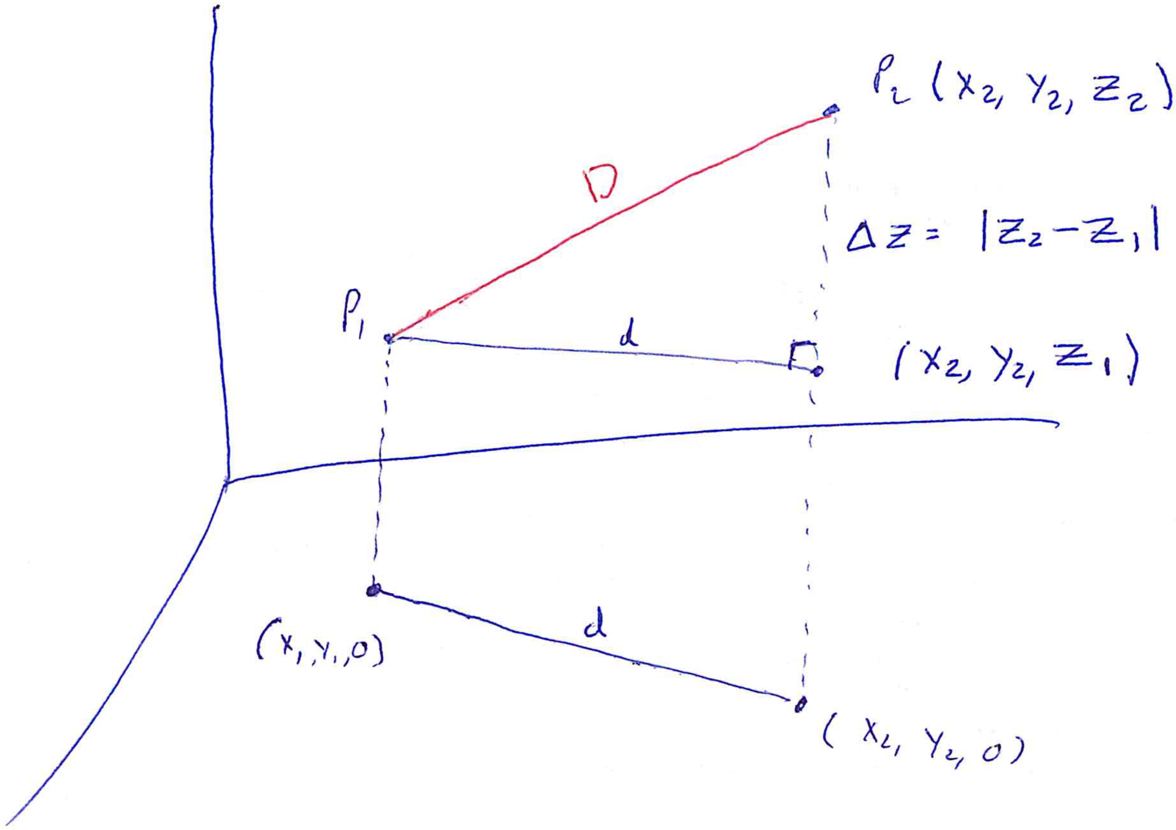
= 3 dimensional space



<u>Distance Formula</u> Let $P_1 = (x_1, y_1, z_1)$, $P_2 = (x_2, y_2, z_2)$.

$$\text{dist}(P_1, P_2) = |P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

It is derived by applying the Pythagorean Theorem twice. (next page.)



$$D^2 = d^2 + (\Delta z)^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

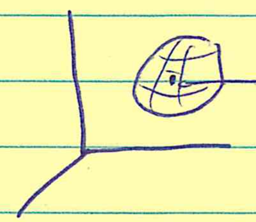
$$D^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Spheres $x^2+y^2+z^2=R^2$ = points in $\mathbb{R}^3$ whose distance to $(0,0,0)$ is R.

For a sphere with center (a,b,c) the formula is

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2.$$



Example Find the center and radius of the sphere given by

$$x^2+y^2+z^2=4x-2y.$$

Then find the intercepts with each axis.

Solution We use completing the square. (see review links if needed.)

$$x^2-4x\ \underline{+4}\ +y^2+2y\ \underline{+1}\ +z^2=\underline{+4\quad+1}$$

$$(x-2)^2+(y+1)^2+z^2=5.$$

Center is $(2,-1,0)$. Radius is $\sqrt{5}$.

(next page)

To find x-intercepts set $y=0$ and $z=0$

$$(x-2)^2 + (0+1)^2 + 0^2 = 5$$

$$(x-2)^2 = 4$$

$$x-2 = \pm 2$$

$$x = 4 \quad x = 0.$$

To find y-intercepts set $x=0$ and $z=0$.

$$(-2)^2 + (y+1)^2 + 0^2 = 5$$

$$(y+1)^2 = 1$$

$$y+1 = \pm 1$$

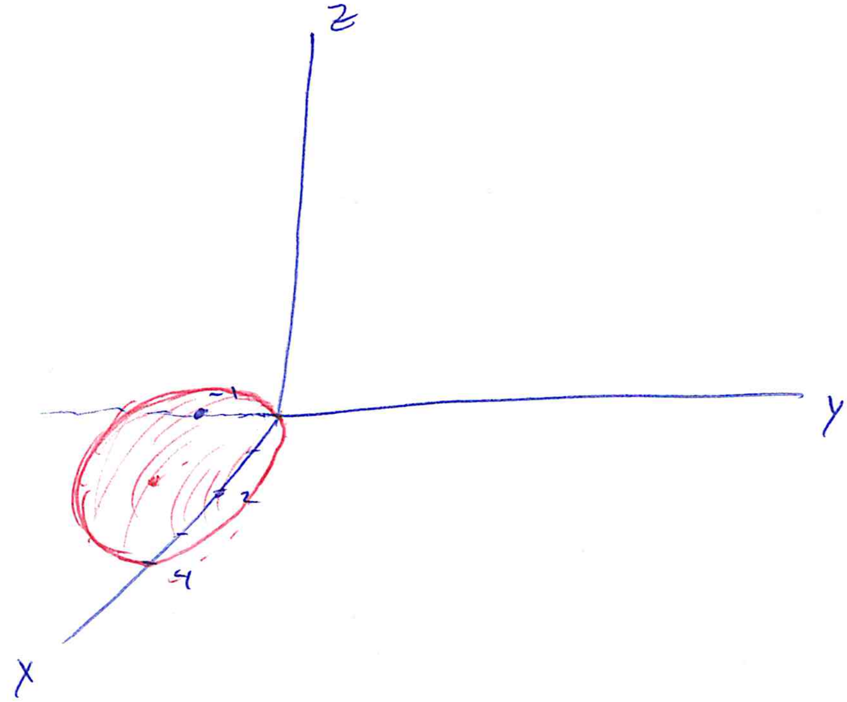
$$y = 0, -2.$$

To find z-intercepts set $x=0$ and $y=0$.

$$(-2)^2 + (1)^2 + z^2 = 5$$

$$z^2 = 0$$

$$z = 0.$$



z
y
x
1
2
4