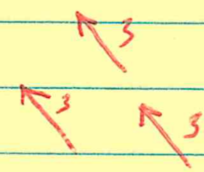
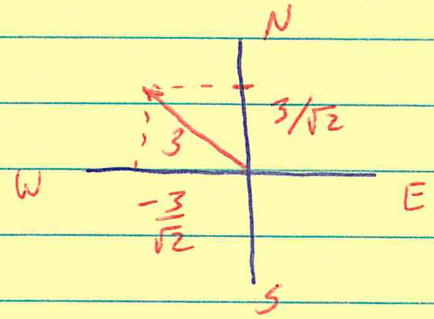
10.2 Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$

Def A vector is a direction and a magnitude.

Ex. Go 3 miles NW.



Components



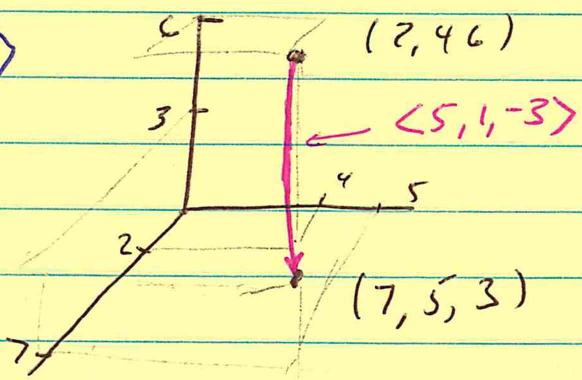
$3\ NW = \left\langle \frac{-3}{\sqrt{2}}, \frac{+3}{\sqrt{2}} \right\rangle$

y-comp.

x-component

Ex The vector from $(2,4,6)$ to $(7,5,3)$ is

$\langle 7-2, 5-4, 3-6\rangle = \langle 5,1,-3\rangle$



Ex What is the magnitude of $\langle 5,1,-3\rangle$?

Answer $|\langle 5,1,-3\rangle| = \sqrt{(5)^2+(1)^2+(-3)^2} = \sqrt{25+1+9}$

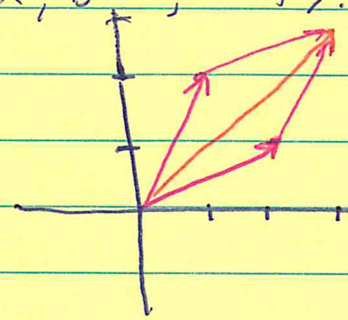
$= \sqrt{35}.$

Vector Addition

Def $\langle a, b\rangle + \langle c, d\rangle = \langle a+c, b+d\rangle$

$\langle a, b, c\rangle + \langle d, e, f\rangle = \langle a+d, b+e, c+f\rangle.$

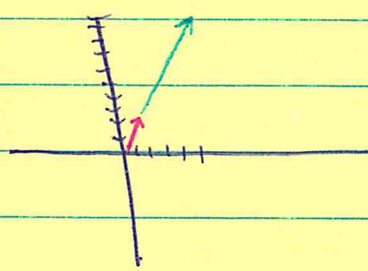
Ex $\langle 1, 2\rangle + \langle 2, 1\rangle = \langle 3, 3\rangle.$

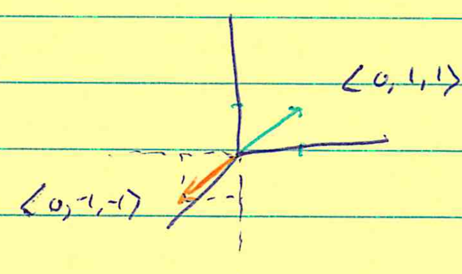


Scalar Multiplication

Def $r\langle a, b\rangle = \langle ra, rb\rangle$, $r\langle a, b, c\rangle = \langle ra, rb, rc\rangle$

Ex's $5\langle 1, 2\rangle = \langle 5, 10\rangle.$ $-\langle 0, 1, 1\rangle = \langle 0, -1, -1\rangle$





Fact $|rv| = |r||v|.$ **Proof:** $|r\langle a, b, c\rangle| = |\langle ra, rb, rc\rangle|$

$= \sqrt{(ra)^2 + (rb)^2 + (rc)^2} = \sqrt{r^2(a^2+b^2+c^2)} = |r|\sqrt{a^2+b^2+c^2}$

$= |r||v|.$

<u>Notation</u> $\mathbf{0} = \langle 0,0 \rangle$ or $\langle 0,0,0 \rangle$.

<u>Properties</u> Let u, v and w be vectors.
Let s and t be real numbers (scalars).

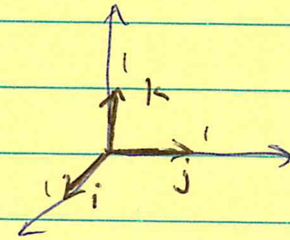
1. $u+v = v+u$
2. $u+(v+w) = (u+v)+w$
3. $u+\mathbf{0} = u$
4. $u+(-u) = \mathbf{0}$
5. $s(u+v) = su+sv$
6. $(s+t)u = su+tu$
7. $s(tu) = (st)u$
8. $1 \cdot u = u$
9. $0 \cdot u = \mathbf{0}$

Pf of 2 for $\mathbb{R}^2$. Let $u = \langle u_1, u_2 \rangle$, $v = \langle v_1, v_2 \rangle$, $w = \langle w_1, w_2 \rangle$

$$\begin{aligned}
u+(v+w) &= \langle u_1, u_2 \rangle + (\langle v_1, v_2 \rangle + \langle w_1, w_2 \rangle) \\
&= \langle u_1, u_2 \rangle + \langle v_1+w_1, v_2+w_2 \rangle \\
&= \langle u_1+(v_1+w_1), u_2+(v_2+w_2) \rangle \\
&= \langle (u_1+v_1)+w_1, (u_2+v_2)+w_2 \rangle \\
&= \langle u_1+v_1, u_2+v_2 \rangle + \langle w_1, w_2 \rangle \\
&= (\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle) + \langle w_1, w_2 \rangle \\
&= (u+v)+w.
\end{aligned}$$

Notation $\mathbf{i} = \langle 1, 0, 0 \rangle$, $\mathbf{j} = \langle 0, 1, 0 \rangle$, $\mathbf{k} = \langle 0, 0, 1 \rangle$.

Ex $\langle 5, 2, 7 \rangle = 5\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$



Def $\mathbf{v}$ is a unit vector if $|\mathbf{v}| = 1$.

Ex Find the unit vector $\mathbf{u}$ with the same direction as $\mathbf{v} = \langle 1, 2, 1 \rangle$.

Solution $|\langle 1, 2, 1 \rangle| = \sqrt{1+4+1} = \sqrt{6}$.

Let $\mathbf{u} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}} \right\rangle$.

You should check that $|\mathbf{u}| = 1$.