For 10.4 Review of Determinants

Def Let A be the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then the determinant of A is

$$\det A = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Ex $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 3\cdot 4 - 2\cdot 1 = 10.$

Application Consider the system of two linear equations with two unknowns:

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

(x and y are the unknowns and a, b, c, d, e and f are given numbers.) The system has a unique solution for x and y if and only if

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0.$$

The proof is covered in MATH 221, but you might give it a try.

Def Let A be a 3×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

Then the determinant of A is

$$\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= aei - afh - bdi + bfg + cdh - ceg.$$

Ex $$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= (1-2) - 2(2-3) + 3(4-3)$$

$$= -1 + 2 + 3 = 4.$$

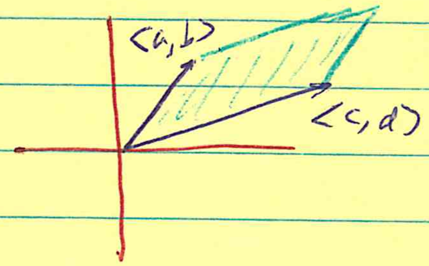
Application The system
$$\begin{aligned} ax + by + cz &= r \\ dx + ey + fz &= s \\ gx + hy + iz &= t \end{aligned}$$

has a unique solution for x, y and z iff

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0.$$

Geometric Applications.

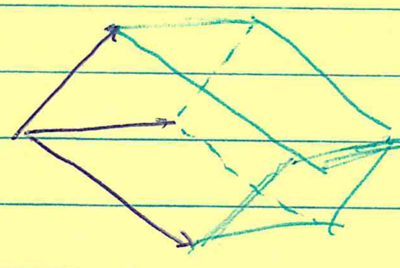
1. Let $\langle a, b \rangle$ and $\langle c, d \rangle$ be vectors in $\mathbb{R}^2$



Then the area of the parallelogram they determine is the absolute value of

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}.$$

2. Let $\langle a, b, c \rangle$, $\langle d, e, f \rangle$ and $\langle g, h, i \rangle$ be vectors in $\mathbb{R}^3$. They define a parallelepiped.



Its volume is the absolute value of

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}.$$