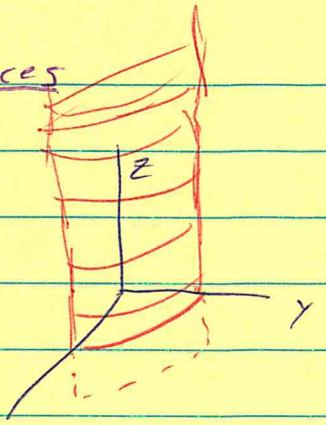
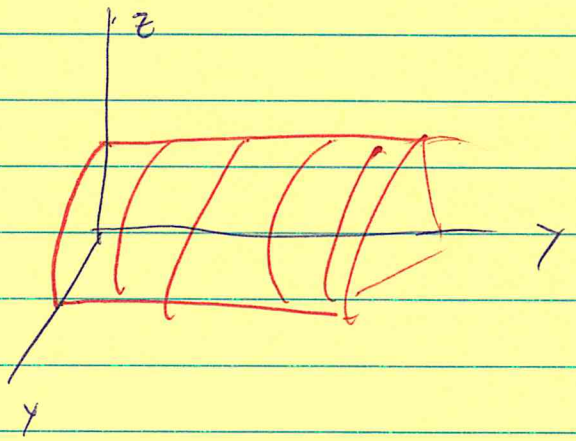
10.6 Cylinders and Quadratic Surfaces

Ex $x^2+y^2=1$ in $\mathbb{R}^3$. (z is free.)

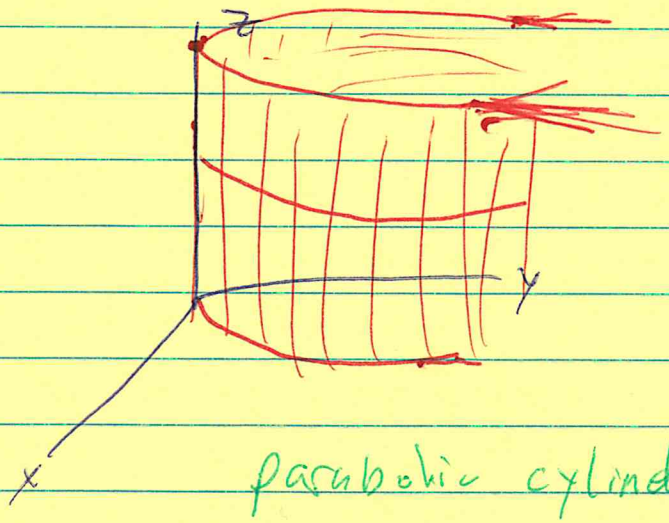


circular cylinder.

Ex $x^2+z^2=1$ in $\mathbb{R}^3$. (y is free)

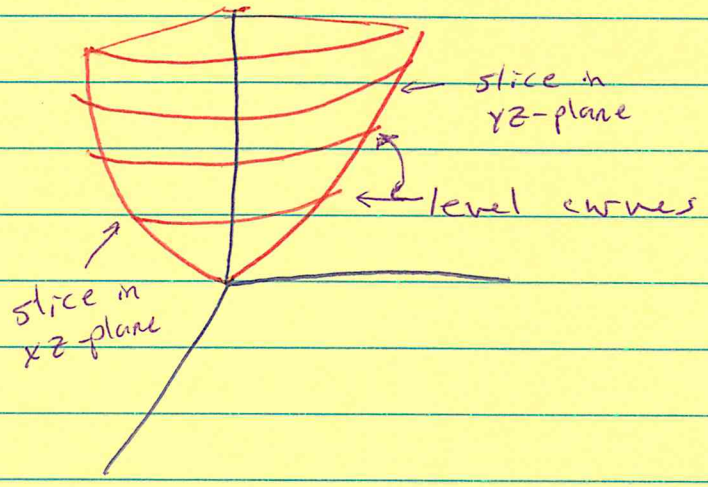


Ex $y=x^2$ in $\mathbb{R}^3$. (z is free)



parabolic cylinder

Ex $z = f(x,y) = x^2 + y^2$ (Paraboloid.)

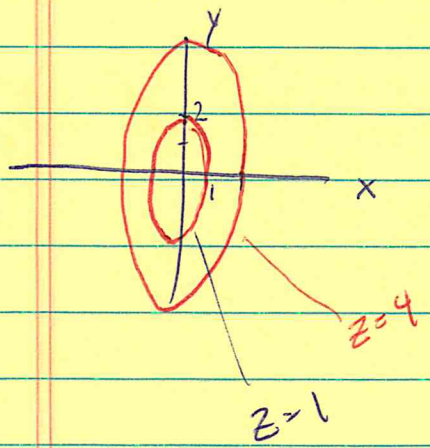


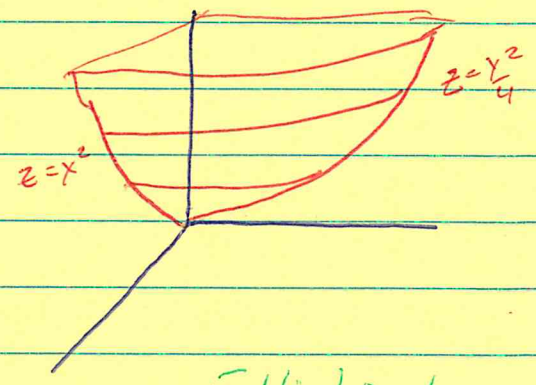
If $y=0$, we get $z = x^2$ as a slice in the xz-plane.
If $x=0$, we get $z = y^2$ as a slice in the yz-plane.

Set $z=1$. Get $x^2+y^2=1$ is a level curve.
Set $z=2$. Get $x^2+y^2=2$ (radius $\sqrt{2}$). etc.

Ex $z = f(x,y) = x^2 + \frac{y^2}{4}$

A paraboloid where the level curves are ellipses.

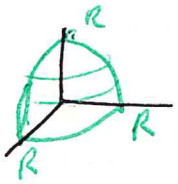




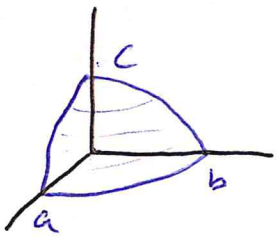
Elliptical Paraboloid.

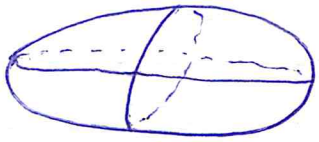
Spheres and Ellipsoids

$$x^2 + y^2 + z^2 = R^2$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$





Length of axes: $2a, 2b, 2c$.

You can move the center:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-j)^2}{c^2} = 1.$$

You can use the disk method to show the volume is

$$V = \frac{4}{3}\pi abc.$$

Example Find the center and the lengths of the 3 principal axes of the ellipsoid given by

$$4x^2 + 9y^2 + 36z^2 - 16x + 18y - 72z + 25 = 0$$

Solution

$$4x^2 - 16x \quad + 9y^2 + 18y \quad + 36z^2 - 72z \quad = -25$$

$$4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) + 36(z^2 - 2z + 1) = -25$$

$+16$

$+9$

$+36$

$$4(x-2)^2 + 9(y+1)^2 + 36(z-1)^2 = 36$$

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} + (z-1)^2 = 1.$$

Center is $(2, -1, 1)$.

Length of axis parallel to x-axis is $2 \times \sqrt{9} = 6$.

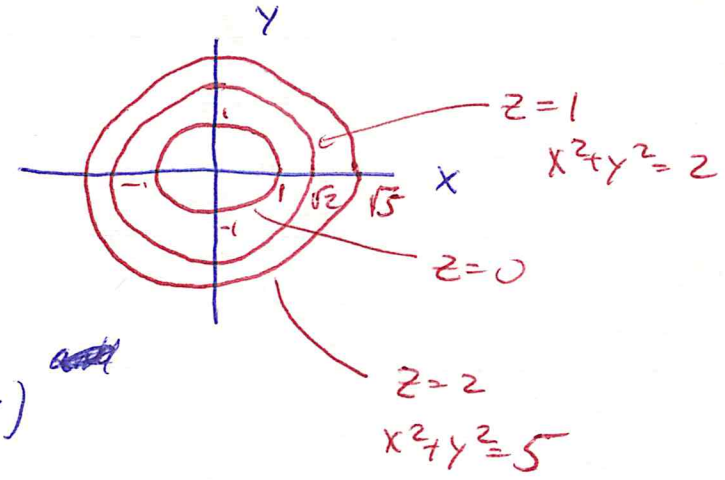
" " " " " Y-axis is $2 \times \sqrt{4} = 4$

" " " " " Z-axis is $2 \times \sqrt{1} = 2$

Hyperboloids.

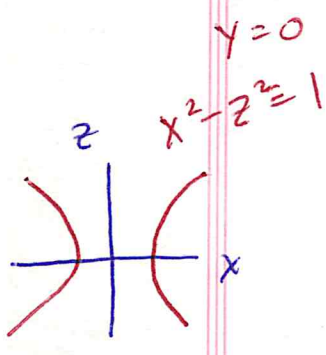
Ex $x^2+y^2-z^2=1$

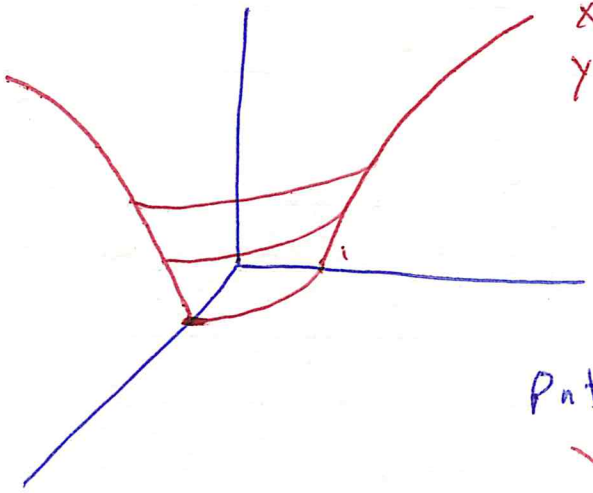
Look at level curves for $z=0$, $z=1$, $z=2$, etc.

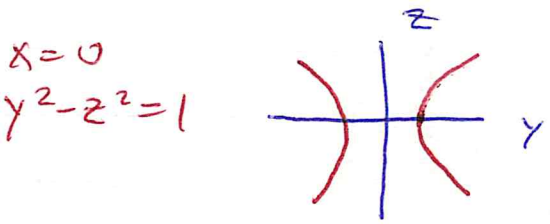


Look at slices for $x=0$ (yz-plane)

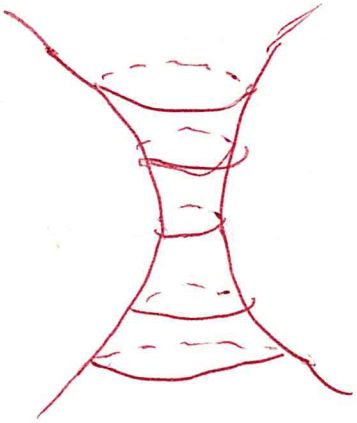
and $y=0$ (xz-plane)





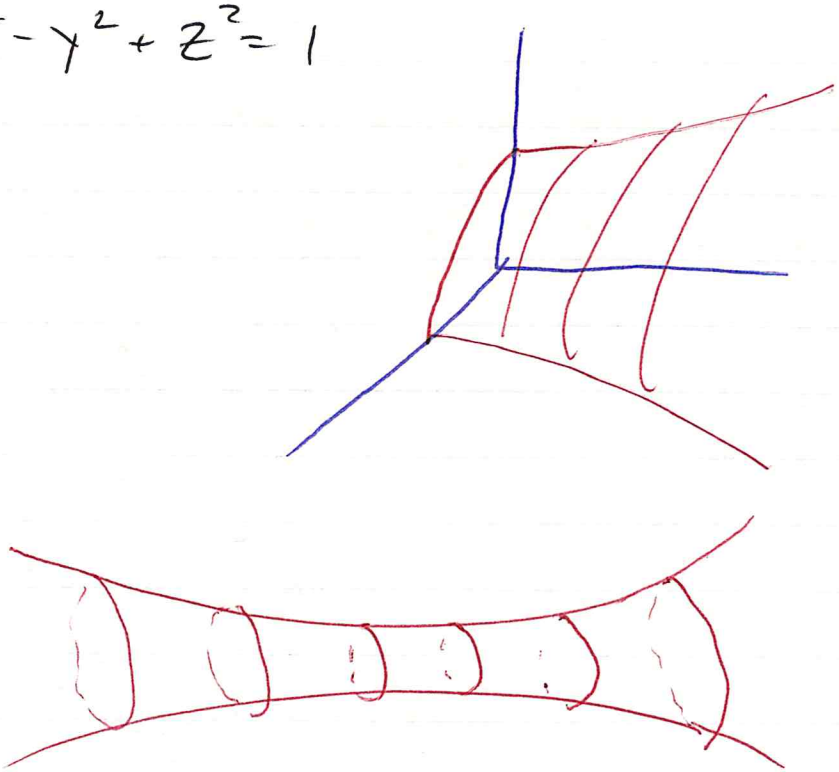


Put it together.



A one sheeted hyperboloid.

Ex $x^2 - y^2 + z^2 = 1$



Ex $-x^2 - y^2 + z^2 = 1$ $\quad x^2+y^2 = -1$

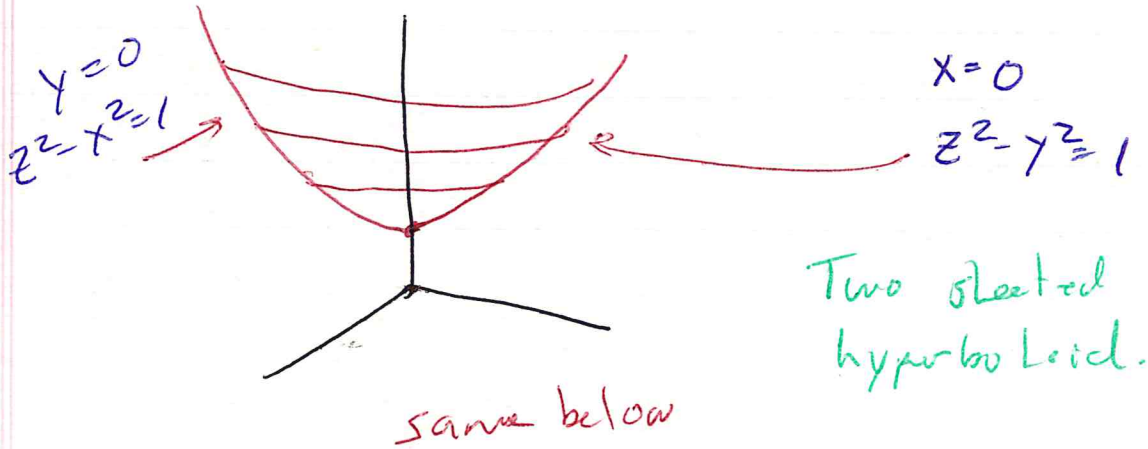
$z = 0 \Rightarrow$ no solutions!

$z = 1 \Rightarrow x^2+y^2 = 0$, $(0,0,1)$ only solution.

$z = 2 \Rightarrow x^2+y^2 = 3$ circle $R = \sqrt{3}$

$z = 3 \Rightarrow x^2+y^2 = 8$ $\quad R = 2\sqrt{2}$

(same for $z = -1, -2, -3$).



Two sheeted hyperboloid.

Ex $-x^2 - y^2 + z^2 = 0$.

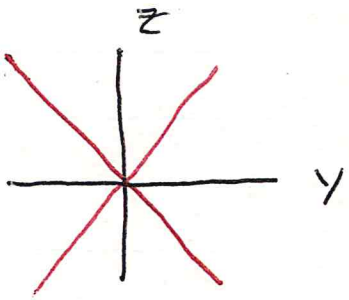
$z = 0 \Rightarrow x^2 + y^2 = 0 \Rightarrow (0,0,0)$ only solution.

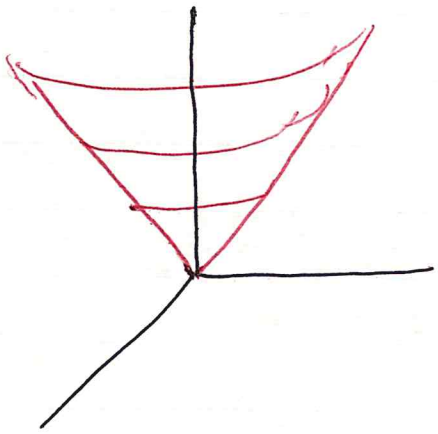
$z = \pm 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow$ circle

$z = \pm 2 \Rightarrow x^2 + y^2 = 4 \Rightarrow$ circle

$x = 0 \Rightarrow y^2 = z^2 \Rightarrow y = \pm z$

$y = 0 \Rightarrow x^2 = z^2 \Rightarrow x = \pm z$





Cone!