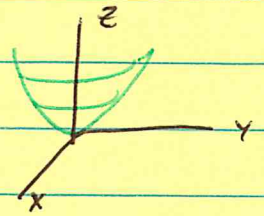
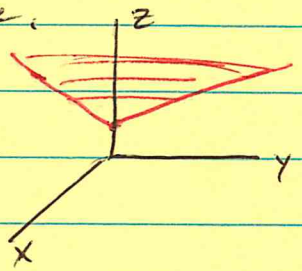
11.1 Functions of Several Variables

Ex $z = x^2 + y^2$ paraboloid - as we have seen before



Ex $z = 2x + y + 1$ linear. a plane.



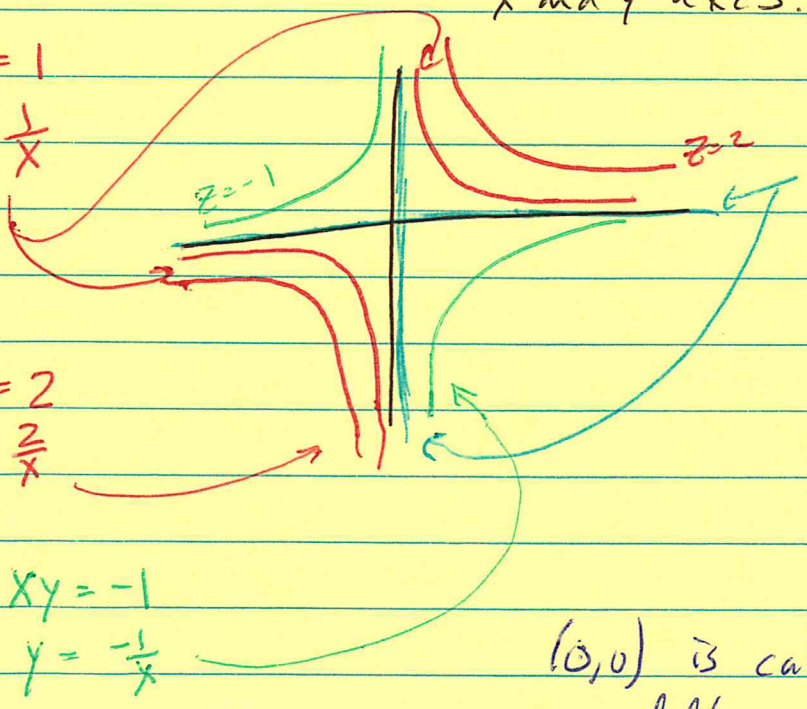
Ex $z = xy$. We will make a contour graph consisting of level curves.

$z = 0$, $xy = 0$ so $x = 0$ or $y = 0$. We get the x and y-axes.

$z = 1$. $xy = 1$ or $y = \frac{1}{x}$

$z = 2$. $xy = 2$ $y = \frac{2}{x}$

$z = -1$. $xy = -1$ $y = -\frac{1}{x}$

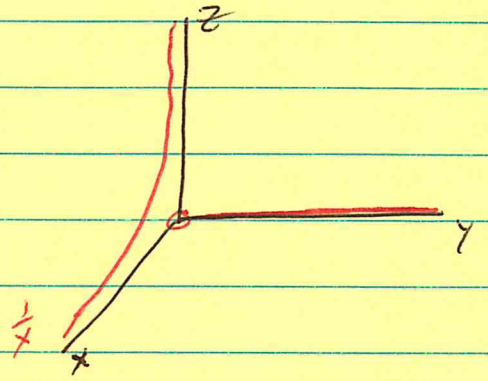


(0,0) is called a saddle point.

Ex Graph $f(x,y) = \frac{x}{x^2+y^2}$. Note: domain is $\mathbb{R}^2 - \{0\}$.

Slices: $x=0$. $f(0,y) = 0$

$y=0$. $f(x,0) = \frac{x}{x^2+0^2} = \frac{1}{x}$.



Level curves: Let $\frac{x}{x^2+y^2} = c$, a constant.

Then $x = cx^2 + cy^2$

or $x^2 - \frac{1}{c}x + y^2 = 0$. These are circles!

$c=1$ $x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4}$

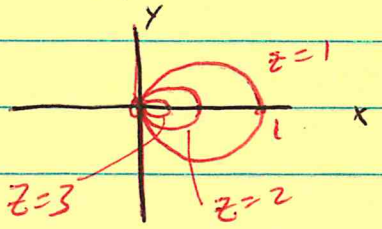
$(x-\frac{1}{2})^2 + y^2 = \frac{1}{4}$ $R = \frac{1}{2}$, center $= (\frac{1}{2}, 0)$.

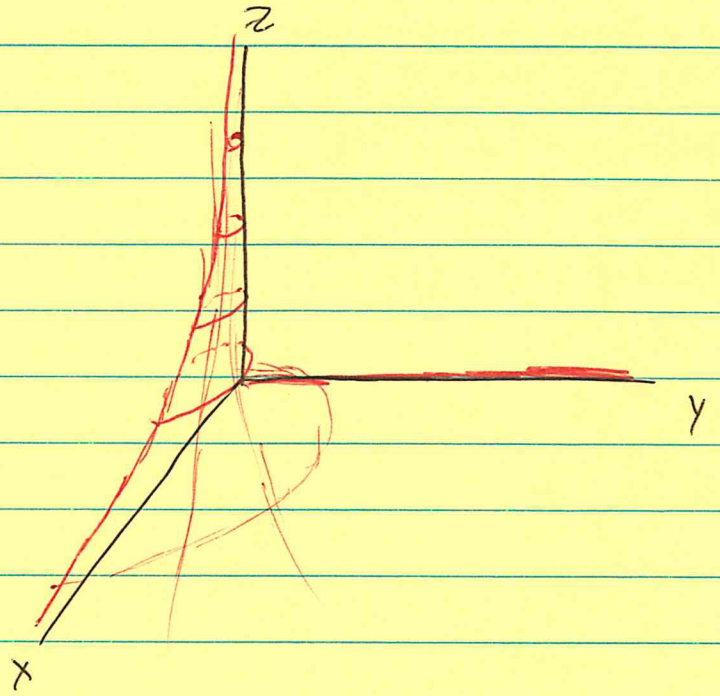
$c=2$ $x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 = 0 + \frac{1}{16}$

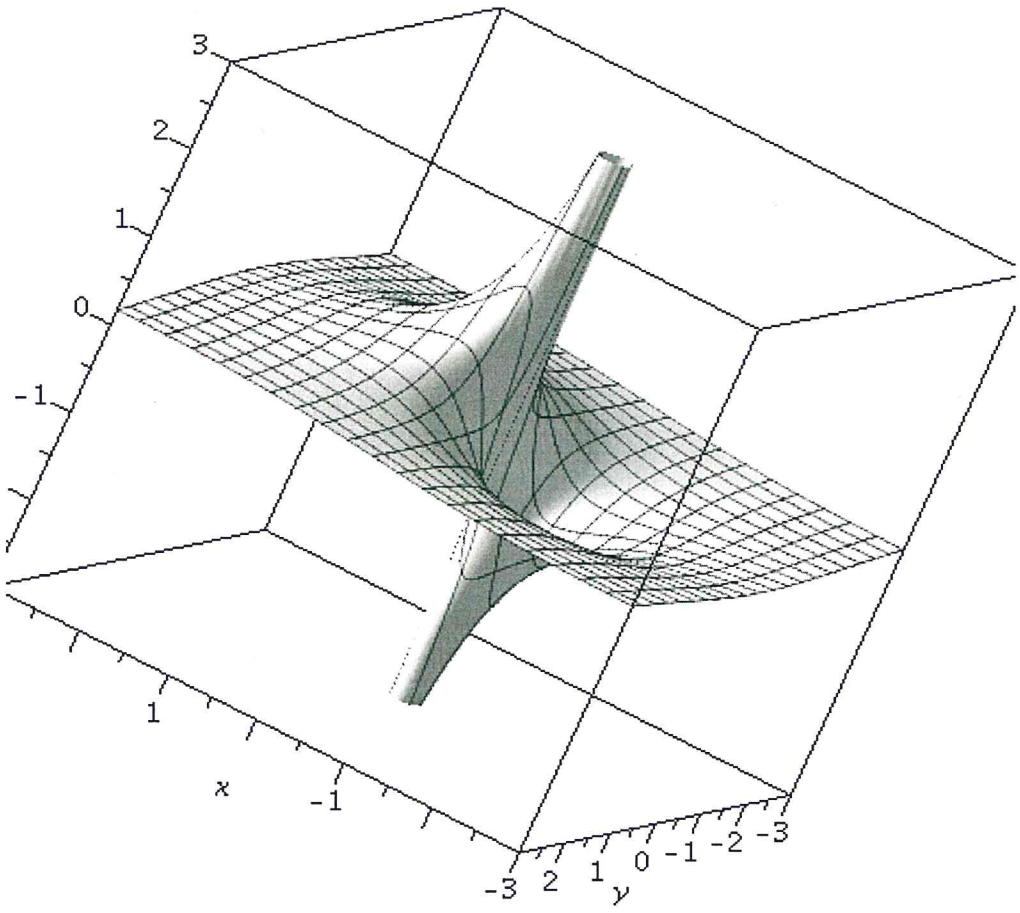
$(x-\frac{1}{4})^2 + y^2 = \frac{1}{16}$ $R = \frac{1}{4}$ center $(\frac{1}{4}, 0)$

$c=3$ $x^2 - \frac{1}{3}x + \frac{1}{36} + y^2 = 0 + \frac{1}{36}$

$(x-\frac{1}{6})^2 + y^2 = \frac{1}{36}$ $R = \frac{1}{6}$ center $(\frac{1}{6}, 0)$



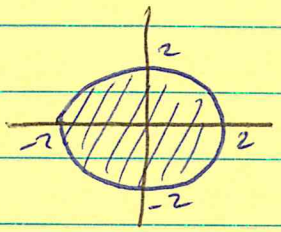

z
y
x


3
2
1
0
-1
1
x
-1
-3
2
1
0
-1
-2
-3
y

Domain Examples

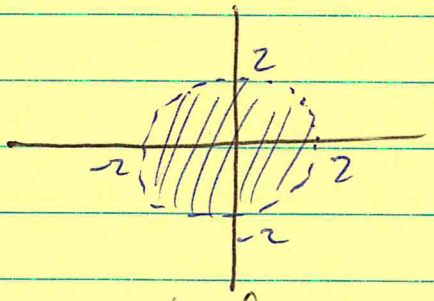
Ex Find the domain of $f(x,y) = \sqrt{4 - x^2 - y^2}$.

Sol We need $4 - x^2 - y^2 \geq 0$. Thus $x^2 + y^2 \leq 4$.
Hence the domain is the disk of radius 2 center $(0,0)$.



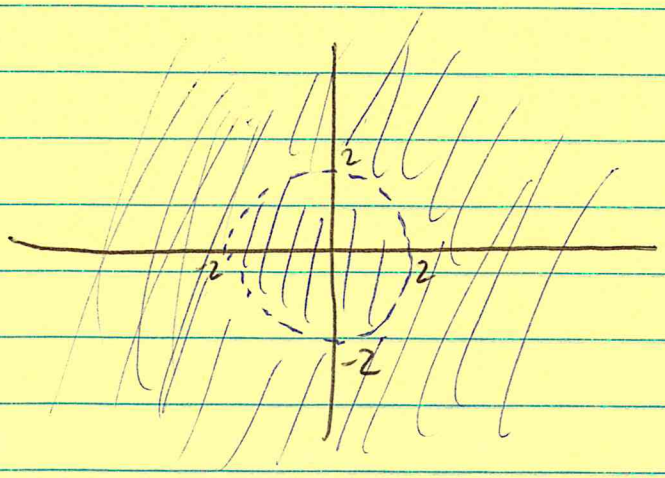
Ex Find the domain of $f(x,y) = \ln(4 - x^2 - y^2)$.

Sol Now we need $4 - x^2 - y^2 > 0$. Thus $x^2 + y^2 < 4$.



Ex Find the domain of $f(x,y) = \dfrac{1}{4 - x^2 - y^2}$.

Sol



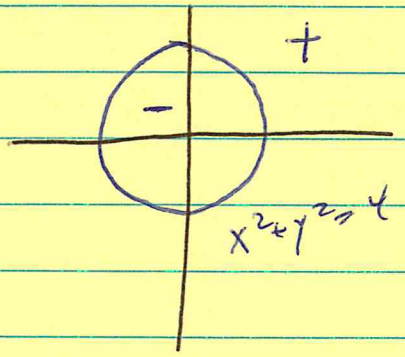
Everywhere except on the circle $x^2 + y^2 = 4$.

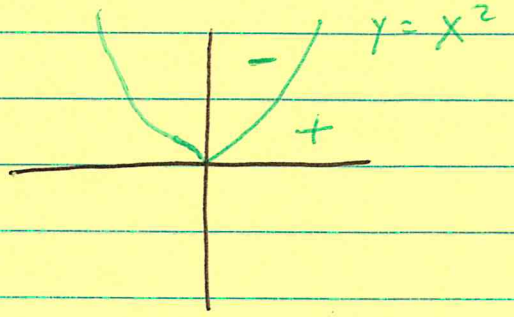
Ex Find the domain of $f(x,y) = \sqrt{(x^2+y^2-4)(x^2-y)}$.

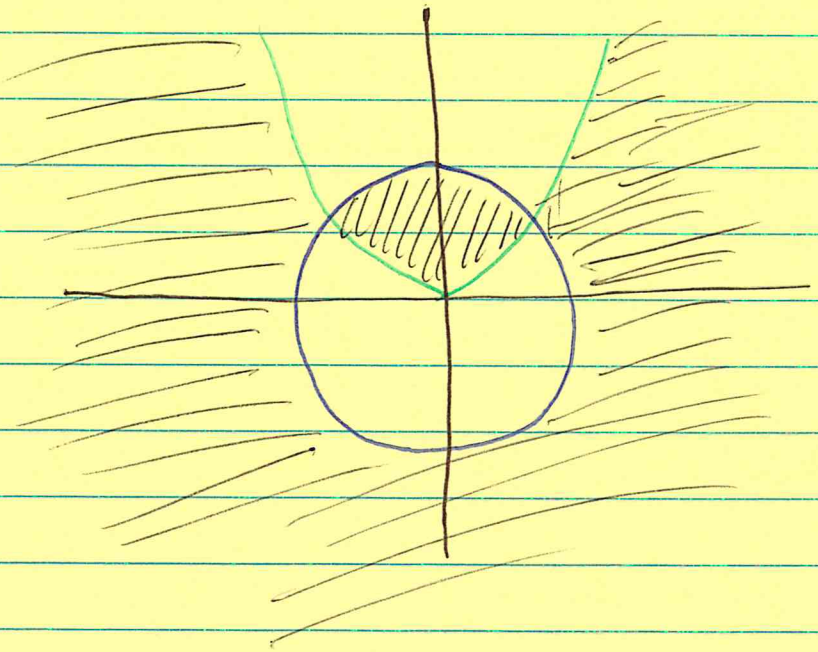
Sol. We need $(x^2+y^2-4)(x^2-y) \geq 0$. Thus,

we need $x^2+y^2-4 \geq 0$ and $x^2-y \geq 0$

or $x^2+y^2-4 \leq 0$ and $x^2-y \leq 0$.



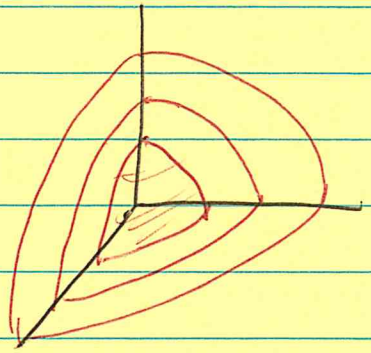




Functions of 3 variables

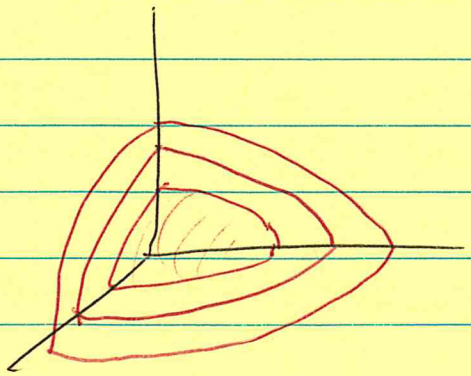
Ex $f(x,y,z) = x^2 + y^2 + z^2$.

Level surfaces are spheres.



Ex $f(x,y,z) = x^2 + \frac{y^2}{4} + z^2$.

Level surfaces are ellipsoids.



Ex $f(x,y,z) = x^2+y^2-z^2$.

Level 0: $x^2+y^2-z^2=0$ is a cone.

Level 1: $x^2+y^2-z^2=1$ is a one sheeted hyperboloid.

Level -1: $x^2+y^2-z^2=-1$

$-x^2-y^2+z^2=1$ is a two sheeted hyperboloid.