Math 251 Practice Test 1 Spring 2002

Name: _____ ID #: _____

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Prove that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$, where $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in R^3.

2. [10 points] Prove that if θ is the angle between two vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}.$$

   Hint: use the law of cosines.

3. [10 points] Let $\mathbf{u} = \langle 2, -2, 1\rangle$ and $\mathbf{v} = \langle -3, 0, 3\rangle$.

   a) Find the projection of $\mathbf{v}$ in the direction of $\mathbf{u}$.

   b) Find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{u}$.

4. [10 points] Consider the points $A(1, 0, 0)$, $B(3, 0, -1)$, and $C(2, 1, -2)$.

   a) Find the area of the triangle with vertices A, B, and C.

   b) Find the equation in standard form of the plane that passes through the points A, B, and C.

5. [10 points] Classify and sketch the surface given by $x - y^2 - 4z^2 = 0$.

6. [10 points] Find $\mathbf{T}(t)$ and then find a set of parametric equations for the tangent line to the helix given by $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$ at the point corresponding to $t = \pi/4$.

7. [10 points] Find the curvature of $\mathbf{r}(t) = e^t \cos t\mathbf{i} + e^t \sin t\mathbf{j} + e^t\mathbf{k}$.

8. [10 points] Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$$

9. [10 points] Integrate $\int \cos 2x \sin 3x\, dx$.

10. [10 points] Find the Taylor series of e^{x^2} centered about $x = 0$.