Math 251 Practice Test 2 Spring 2002

Name: _____ ID #: _____

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Let $f(x, y, z) = x^2y + zxy + z^2 + 2x$. What is the rate of change of the value of f at the point $(1, 2, 1)$ along the line $\mathbf{r}(t) = \langle 3, 2, -1\rangle\, t + \langle 1, 2, 1\rangle$ with respect to t (time)?

2. [10 points] Let $f(x, y) = x^3y^2 + 3xy$. Find the equation in standard form of the plane tangent to the surface $z = f(x, y)$ at the point $(1, 2)$. Find a unit normal vector for this plane.

3. [10 points] Minimize $f(x, y) = x^2 - y^2$ subject to the constraint $x - 2y + 6 = 0$.

4. [10 points] Find dy/dx, given $y^3 + y^2 - 5y - x^2 + 4 = 0$.

5. [10 points] Find and classify the relative extrema of $f(x, y) = x^2y^2$.

6. [10 points] Find the volume inside the cylinder $x^2 + y^2 = 1$, above the xy-plane and below the plane $2x + 3y - z = -6$.

7. [10 points] Let $r = 2\sin 3\theta$. Graph this in polar coordinates. Find the total area enclosed.

8. [10 points] Set up but do not evaluate the integral needed to find the *radius of gyration* about the y-axis for the lamina corresponding to the region given by: $0 \le y \le \sin x$, $0 \le x \le \pi$, where the density function is $\rho(x, y) = x$.

9. [10 points] Find the mass and center of mass of the lamina bounded by $y = 1/(1 + x^2)$, $x = 1$ and $x = -1$, with density function $\rho = 3$.

10. [10 points]

    a) Prove that $(x^2)' = 2x$.

    b) Compute $\int_0^1 xe^x\, dx$.