

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Compute the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + xy^2}{x^2 + y^2}$.
2. [15 points] Let $f(x, y, z) = x^3 + y^2z + 2$. Find the rate of change of the value of f in the direction $\mathbf{v} = \langle 1, 1, 1 \rangle$ at the point $(1, 2, 3)$.
3. [15 points] Let $z = f(x, y)$, $x = r \cos \theta$, and $y = r \sin \theta$.
 - a) Use the Chain Rule to write $\partial z / \partial r$ and $\partial z / \partial \theta$.
 - b) Verify that $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$.
4. [10 points] Find and classify any critical points of $f(x, y) = (x - 1)^2(y + 4)^2$.
5. [10 points] Sketch the region of integration and evaluate the double integral

$$\int_0^1 \int_y^{\sqrt{y}} x^2 y^2 dx dy.$$

6. [15 points] Find the center of mass of the lamina corresponding to the parabolic region $0 \leq y \leq 4 - x^2$ where the density at the point (x, y) is proportional to the distance between (x, y) and the x -axis. See figure on board.
7. [15 points] Graph $r = 3 \cos 2\theta$ in polar coordinates, for $0 \leq \theta \leq 2\pi$. Find the total area enclosed.
8. [5 points] Find the Taylor series of e^{x^2} centered about $x = 0$.
9. [5 points] Let $f(x) = 3/x + 7$. Find $f^{-1}(x)$, the inverse function of f .