

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Let $\mathbf{u} = \langle 1, 1, 1 \rangle$, $\mathbf{v} = \langle 0, 2, 2 \rangle$, and $\mathbf{w} = \langle -1, 0, 3 \rangle$.
 - a) Find the volume of the parallelepiped determined by \mathbf{u} , \mathbf{v} , and \mathbf{w} .
 - b) Find the projection of \mathbf{u} onto \mathbf{v} .
 - c) Find the angle between \mathbf{v} and \mathbf{w} , in degrees.
2. [10 points] Find the tangential and normal components of acceleration for a particle that moves according to $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$.
3. [10 points] Let $\mathbf{r}(t) = \langle \cos \pi t, \sin \pi t, t \rangle$. Graph $\mathbf{r}(t)$ for $0 \leq t \leq 2$ and find the total length traced out. *Answer:* $2\sqrt{\pi^2 + 1}$
4. [10 points] Find the minimum and maximum value of the function $f(x, y) = 4x^2 + 9y^2$ when x and y must satisfy $x^2 + y^2 = 1$. *Answer:* Max is 9. Min is 4.
5. [10 points] The sphere of radius 2 has density proportional to the distance to the z -axis. Find its mass and center of mass. *Answer:* Mass is $8k\pi^2/3$. C.M. is obvious.
6. [10 points] Find the area bounded by one loop of $r = 2 \cos 2\theta$. *Answer:* π
7. [10 points] Let $\mathbf{F} = k\mathbf{r}/\|\mathbf{r}\|^3$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find the work done by \mathbf{F} in moving a particle along the straight line segment C from $(0, 4, 0)$ to $(0, 4, 3)$. *Answer:* $k/20$
8. [10 points] Let $\mathbf{F} = \langle x, y, 3z \rangle$. Find the flux of \mathbf{F} up through the portion of the plane $2x + 3y + z = 12$ in the positive octant. *Answer:* 132
9. [10 points] Find the volume bounded by the plane $z = y + 2$ and the paraboloid $z = x^2 + y^2$. *Answer:* 7.5π
10. [10 points] Let $\mathbf{r} = \langle x, y, z \rangle$. Prove that $\nabla \ln \|\mathbf{r}\| = \mathbf{r}/\|\mathbf{r}\|^2$.
11. [10 points] (a) Compute $\oint_C (x + y^2) dx + (y + x^2) dy$ where C is the boundary of the square $[-1, 1] \times [-1, 1]$ counterclockwise. (b) Repeat where C is the boundary of the square $[0, 1] \times [0, 1]$ counterclockwise. *Answers:* 0, 2.
12. [10 points] Let $\mathbf{F} = yz^3\mathbf{i} + (2z - x^4)\mathbf{j} + x \sin(y)\mathbf{k}$. Let S be the unit sphere centered at $(0, 0, 0)$. Find the flux of \mathbf{F} out through S . *Answer:* 0
13. [10 points] Let $\mathbf{F} = \langle 3z, 5x, -2y \rangle$. Let C be the ellipse formed from the intersection of the plane $z = y + 3$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above. Compute $\oint_C \mathbf{F} \cdot \mathbf{T} ds$ *Answer:* 2π .