Math 251 Practice Final Exam Spring 2002

SCIENTIFIC CALCULATORS ONLY

1. [10 points] Let $\mathbf{u} = \langle 1, 1, 1\rangle$, $\mathbf{v} = \langle 0, 2, 2\rangle$, and $\mathbf{w} = \langle -1, 0, 3\rangle$.
   a) Find the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.
   b) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$.
   c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$, in degrees.
2. [10 points] Find the tangential and normal components of acceleration for a particle that moves according to $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \frac{1}{3}t^3\mathbf{k}$.
3. [10 points] Let $\mathbf{r}(t) = \langle \cos \pi t, \sin \pi t, t\rangle$. Graph $\mathbf{r}(t)$ for $0 \le t \le 2$ and find the total length traced out. *Answer:* $2\sqrt{\pi^2 + 1}$
4. [10 points] Find the minimum and maximum value of the function $f(x, y) = 4x^2 + 9y^2$ when x and y must satisfy $x^2 + y^2 = 1$. *Answer:* Max is 9. Min is 4.
5. [10 points] The sphere of radius 2 has density proportional to the distance to the z-axis. Find its mass and center of mass. Hint: $r = \rho \sin \phi$. *Answer:* Mass is $4k\pi^2$. C.M. is obvious.
6. [10 points] Find the area bounded by one loop of $r = 2 \cos 2\theta$. *Answer:* $\pi/2$
7. [10 points] Let $\mathbf{F} = k\mathbf{r}/||\mathbf{r}||^3$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find the work done by $\mathbf{F}$ in moving a particle along the straight line segment C from (0,4,0) to (0,4,3). *Answer:* $k/20$
8. [10 points] Let $\mathbf{F} = \langle x, y, 3z\rangle$. Find the flux of $\mathbf{F}$ up through the proportion of the plane $2x + 3y + z = 12$ in the positive octant. *Answer:* 240
9. [10 points] Find the volume bounded by the plane $z = y + 2$ and the paraboloid $z = x^2 + y^2$. *Answer:* 7.5π
10. [10 points] Let $\mathbf{r} = \langle x, y, z\rangle$. Prove that $\triangledown \ln ||\mathbf{r}|| = \mathbf{r}/||\mathbf{r}||^2$.
11. [10 points] (a) Compute $\oint_C (x + y^2)\, dx + (y + x^2)\, dy$ where C is the boundary of the square $[-1, 1] \times [-1, 1]$ counterclockwise. (b) Repeat where C is the boundary of the square $[0, 1] \times [0, 1]$ counterclockwise. *Answers:* 0, 2.
12. [10 points] Let $\mathbf{F} = yz^3\mathbf{i} + (2z - x^4)\mathbf{j} + x \sin(y)\mathbf{k}$. Let S be the unit sphere centered at (0,0,0). Find the flux of $\mathbf{F}$ out through S. *Answer:* 0
13. [10 points] Let $\mathbf{F} = \langle 3z, 5x, -2y\rangle$. Let C be the ellipse formed from the intersection of of the plane $z = y + 3$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above. Compute $\oint_C \mathbf{F} \cdot \mathbf{T} ds$ *Answer:* 2π.