Math 251
Practice Test A For Chapter 10

1. [10 points] Prove that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$, where $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in R^3.

2. [10 points] Prove that if θ is the angle between two vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}.$$

   Hint: use the law of cosines.

3. [15 points] Let $\mathbf{u} = \langle 2, -2, 1\rangle$ and $\mathbf{v} = \langle -3, 0, 3\rangle$.

   a. Find the projection of $\mathbf{v}$ in the direction of $\mathbf{u}$.

   b. Find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{u}$.

   c. Find the angle between $\mathbf{u}$ and $\mathbf{v}$ in degrees.

4. [10 points] Consider the points $A(1, 0, 0)$, $B(3, 0, -1)$, and $C(2, 1, -2)$.

   a. Find the area of the triangle with vertices A, B, and C.

   b. Find the equation in standard form of the plane that passes through the points A, B, and C.

5. [10 points] Consider the surface given by $x = y^2 + 4z^2$.

   a. Graph the slice for $x = 1$.

   b. Graph the slice for $y = 0$.

   c. Which type of surface is this?

6. [15 points] Let $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$. It determines a helix in R^3.

   a. What is the speed when $t = \pi/4$?

   b. Compute $\mathbf{T}(\pi/4)$.

   c. Find a set of parametric equations for the tangent line to this helix at the point corresponding to $t = \pi/4$.

7. [10 points] Find the curvature of $\mathbf{r}(t) = e^t\cos t\mathbf{i} + e^t\sin t\mathbf{j} + e^t\mathbf{k}$.

8. [10 points] Let $f(x) = x^3$.

   a. Find the curvature $\kappa(x)$ of $f(x)$.

   b. Sketch the graph of $y = \kappa(x)$.