Math 251
Practice Test B For Chapter 10

1. [10 points] Suppose that $\mathbf{r}(t)$ is a spacecurve is $\mathbb{R}^3$ such that its magnitude is a constant, $|r(t)| = C$. Prove that $\mathbf{r}(t)$ is always perpendictular to $\mathbf{r}'(t)$.

2. [5 points] Find the volume of the parallelepiped determined by the vectors $\langle 1, 0, 1\rangle$, $\langle 1, 0, 0\rangle$, and $\langle 1, 1, 1\rangle$.

3. [15 points] Find the distance between the point $Q(1, 5, -4)$ and the plane given by $3x - y + 2z = 6$. Hint: Pick some point A on the plane. Then project the vector $\overrightarrow{QA}$ onto a normal vector for the plane.

4. [10 points] Consider the surface given by $4x^2 - 3y^2 + 12z^2 + 12 = 0$.
   a. Show that the slice for $y = 0$ is the empty set.
   b. Which type of surface is it?

5. [10 points] Consider the parametric equations $x(t) = 2\sin^2 t$ and $y(t) = 2\cos^2 t$. Sketch its graph in the xy-plane.

6. [10 points] A particle of mass 1 is subjected to a force given by

$$\mathbf{F}(t) = \left\langle 1, t, e^t \right\rangle.$$

   At time $t = 0$ the velecity vector is $\mathbf{0}$ (the zero vector). Set up an integral to find how far the particle travels in the next 10 seconds. **Do not solve it.**

7. [10 points] Sketch the level curves of $z = e^{xy}$ for $z = 0$, 1, and 2.

8. [10 points] Find an equation for the line in $\mathbb{R}^3$ going through the points $(1, -2, 3)$ and $(7, 2, 1)$.

9. [10 points] Find a vector function that represents the curve formed by the intersection of the cylinder given by $x^2 + y^2 = 16$ and the plane give by $x + z = 5$.

10. [10 points] Let $\mathbf{r}(t) = \langle 3t^2, -2t^2, t^2\rangle$. Find the arc length from $t = 0$ to $t = 1$. (Answer is $\sqrt{14}$.)