

Math 251
Practice Test A for Chapter 11

1. Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2+y^2}$$

2. Let $f(x, y, z) = \cos(xy) \tan(xz^3)$. Find f_{xy} and f_{zz} .
3. Let $f(x, y, z) = x^2y + zxy + z^2 + 2x$. What is the rate of change of the value of f with respect to t (time) at the point $(1, 2, 1)$ as we move according to $\mathbf{r}(t) = \langle 3, 2, -1 \rangle t + \langle 1, 2, 1 \rangle$?
4. Let $f(x, y) = x^3y^2 - 3xy$. Find the equation of the plane tangent to the surface $z = f(x, y)$ at the point $(1, 2)$. Express your answer in the form $Ax + By + Cz = D$.
5. The equation $x^2 + 3y^2 - xz^2 = 3$ determines a surface in 3-dimensional space. Find an equation for the tangent plane to this surface at the point $(1, 1, 1)$. Express your answer in the form $Ax + By + Cz = D$.
6. Minimize $f(x, y) = x^2 - y^2$ subject to the constraint $x - 2y + 6 = 0$.
7. Find the minimum and maximum values of $f(x, y, z) = 3x + 2y + z$ constrained to be on the unit sphere centered at the origin.
8. Find $\frac{\partial z}{\partial x}$ where $x^3 + y^3 + z^3 + 6xyz = 9$ at the point $(1, 1, 1)$.
9. Find and classify the relative extrema and saddle points of $f(x, y) = x^3 - 2xy + y^2$.
10. Show that $q(x, y) = e^{xy}$ satisfies

$$q_{xx} + q_{yy} = (x^2 + y^2)q.$$