

Math 251  
Practice Test B for Chapter 11

1. Show that the following limit does not exist.

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy + xz + yz}{x^2 + y^2 + z^2}$$

2. Let  $f(x, y, z) = \sin(3x + yz)$ . Find  $f_{xyz}$ .

3. Let  $u(x, y) = e^{2x} \sin(3y)$ . Show that

$$9u_{xx} + 4u_{yy} = 0.$$

4. Find the directional derivative of  $f(x, y, z) = xy^2 - 3z^2$  in the direction  $\langle 1, -1, 2 \rangle$  at  $(1, 1, 2)$ .

5. Let  $z = xe^y + x^2$ . Find an equation for the plane tangent to this surface when  $x = 1$  and  $y = 0$ . Express your answer in the form  $Ax + By + Cz = D$ .

6. A process is known to satisfy the condition  $x^2y + y^3 + xz = 3$ . Suppose we are at the point  $(1, 1, 1)$  on this surface and that  $x$  is held fixed. If we change  $y$  a little what will be the rate of change of  $z$ ? That is, compute

$$\frac{\partial z}{\partial y}(1, 1, 1).$$

7. Find and classify the critical points of  $f(x, y) = 3x - x^3 - 3xy^2$ .
8. Let  $f(x, y, z) = x^4 + y^4 + z^4$ . Find the location of the minimum value of  $f$  subject to the constraint  $4x + 4y - 4z = 1$ .
9. Let  $f(x, y) = 2x^3 + xy^2$ . Find the directional derivative of  $f$  at  $(1, 2)$  going at a  $45^\circ$  angle from the positive  $x$ -direction.
10. Let  $f(x, y) = y^2 - 4xy + 4x^2$ . Find the minimum and maximum values of  $f$  when restricted to the unit circle centered at  $(0, 0)$ .