

Math 251  
Practice Problems for Chapter 12 Test

1. Evaluate  $\int_0^\pi \int_1^2 y \sin(xy) dx dy$ .  
*Answer:* 0.
2. Evaluate  $\int_0^1 \int_x^1 \sin(y^2) dy dx$ . However, to do this you must change the order of the integration. (a) First draw the region. (b) Then set up the new integral. (c) Finally, evaluate it.  
*Answer:*  $(1 - \cos(1))/2$ .
3. Draw the region bounded by  $y = x$ ,  $y = 4$ , and  $x = 0$ . Compute the integral of  $y^2 e^{xy}$  over this region.  
*Answer:*  $(e^{16} - 17)/2$ .
4. Let  $R$  be the 3-dimensional region bounded by  $x^2 + y^2 = 1$ ,  $z = x^2 + y^2 + 10$  and  $z = 2x + y - 7$ . Find the volume of  $R$ .  
*Answer:*  $35\pi/2$ .
5. Set up but do not evaluate the integrals needed to find the *radius of gyration* about the  $x$ -axis for the region given by:  $0 \leq y \leq \sin x$ ,  $0 \leq x \leq \pi$ , where the density function is  $\rho(x, y) = x$ .
6. Find the center of mass of the region bounded by  $y = 1/(1 + x^2)$ ,  $x = 1$  and  $x = -1$ , with density function  $\rho = k$ .  
Hint. The substitution  $x = \tan \theta$  may be helpful.  
*Answer:* Approximately,  $(0, 0.4091549430)$
7. Find the center of mass of the hemispherical region below  $z = \sqrt{25 - x^2 - y^2}$  and above the  $xy$ -plane with density function  $\rho = z + 1$ .  
*Answer:* Mass is  $\approx 752.673$ . C.M. is  $\approx (0, 0, 2.391)$ .
8. Find the rotational inertia (also called the moment of inertia) with respect to a diameter of a solid sphere of radius  $R$  and uniform density  $k$ .  
*Answer:*  $\frac{8\pi k R^5}{15}$ .
9. Consider the region bounded by  $r = \sin z$ ,  $0 \leq z \leq \pi$  using cylindrical coordinates. If it has uniform density  $k$ , find the moment of inertia and the radius of gyration with respect to the  $z$ -axis.  
*Answer:*  $I_z = 3\pi^2 k/16$ ,  $R_G = \sqrt{3/8}$ .
10. Consider a tetrahedron in the first octant, bounded by the coordinate planes and a plane which has  $x$ -intercept  $a$ ,  $y$ -intercept  $b$  and  $z$ -intercept  $c$ . ( $a$ ,  $b$  and  $c$  are positive.) Show that the volume is  $abc/6$ .

11. Consider a region bounded below by the  $xy$ -plane, above by the surface  $z = 10 - 3x^2 + y^2$  and inside the cylinder  $x^2 + y^2 = 1$ . Find the volume.  
*Answer.*  $19\pi/2$ .
12. Consider the region inside the sphere  $x^2 + y^2 + z^2 = 9$ , but outside the cone  $z = r$ . If the density is uniform,  $K$ , find the moment of inertia with respect to the  $z$ -axis.  
*Answer.*  $81K\sqrt{2}/2$ .
13. Consider the region inside the sphere  $x^2 + y^2 + z^2 = 10$ , but outside the cylinder  $x^2 + y^2 = 1$ . If the density is uniform,  $K$ , find the mass.  
*Answer.*  $108K\pi/3$ .