

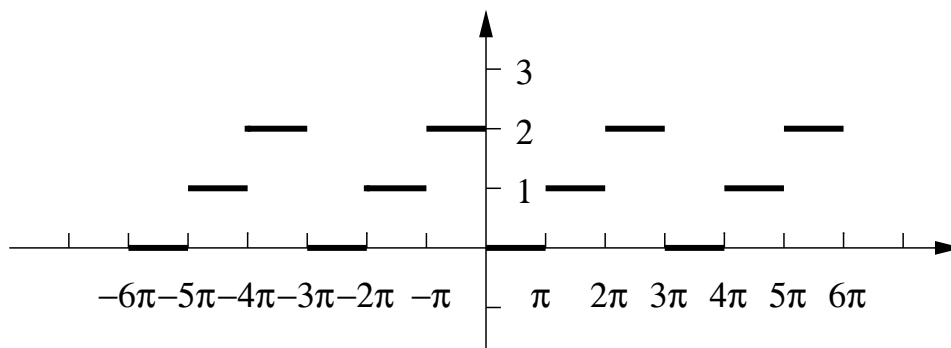
Each problem is worth 20 points.

NO CALCULATORS

1.
 - a) A mass of 5 kg stretches a spring 10 cm. What is the spring constant K in Newtons/meter? (Recall $g \approx 9.8 \text{ m/s}^2$.)
 - b) The drag on the system is 2 Newtons when the speed is 4 cm/s. What is the damping coefficient γ in Newtons/(m/s)?
 - c) The external force on the system is $5\cos(2t)$. What is the equation for the system?
 - d) The initial conditions are $u(0) = u'(0) = 0$. Find the steady-state solution.
2. Let $y'' + (x-1)y' + 3y = 0$. Use the series method, centered about $x_0 = 1$, to find the general solution. You must find a recursive formula for a_n .
3. Let $2y'' + y' + \sin(x)y = 0$. Let $y = \sum_{n=0}^{\infty} a_n x^n$ be the solution. Let $y(0) = 1$ and $y'(0) = 2$. Find a_2 , a_3 and a_4 .
4. Find the solution of the heat conduction problem

$$\begin{aligned}
 100u_{xx} &= u_t, & 0 < x < 1, & & t > 0 \\
 u(0, t) &= 0, & u(1, t) &= 0 & t > 0 \\
 u(x, 0) &= \sin(2\pi x) - 2\sin(5\pi x), & 0 \leq x \leq 1.
 \end{aligned}$$

5. Let $f(x)$ be a periodic function defined by the graph below.



- a. Find a_0 .
- b. Find b_3 .