Math 305 Test 3 Spring 1999

Name: ______________________ ID #: ______________________

1. [4 points] What is the radius of convergence of the Taylor series of $1/(1+x^2)$, centered about $x = 2$? [Note: you do not need to find the Taylor series.]

2. [6 points] Let $f(x)$ and $g(x)$ be even functions and let $h(x)$ be an odd function. Prove the statements below.

   a. $f(x)h(x)$ is odd.

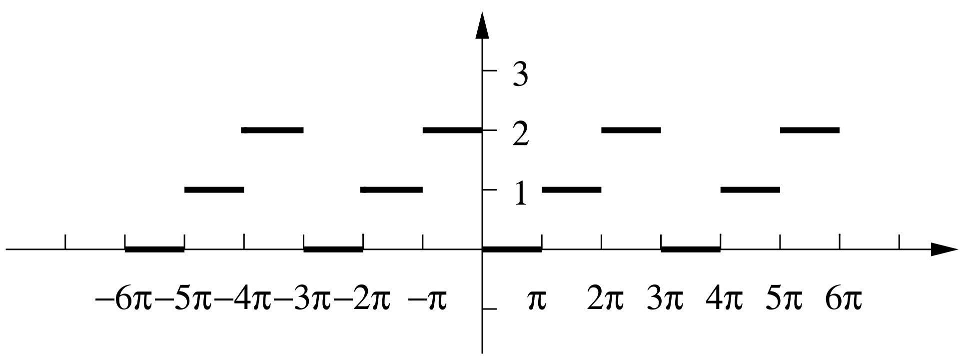
   b. $f(x) + g(x)$ is even.

   c. $h(g(x))$ is even.

3. [20 points] Let $3y'' - (\ln x + 1)y' - xy = 0$. Let $y = \sum_{n=0}^{\infty} a_n x^n$ be the solution. Let $y(0) = 1$ and $y'(0) = 2$. Find a_2, a_3 and a_4.

4. [20 points] Let $y'' + (x-2)y' + (x-2)y = 0$. Use the series method, centered about $x_0 = 2$, to find the general solution. You must find a recursive formula for a_n.

5. [20 points] Let $f(x)$ be a periodic function defined by the graph below.



   a. Find a_0.

   b. Find b_3.

6. [20 points] Consider the partial differential equation

$$U_{xx} - U_{xt} - U_t = 0.$$

Suppose that there is a solution of the form $U(x,t) = X(x)T(t)$. Show that $X(x)$ and $T(t)$ must satisfy the ordinary differential equations below:

$$X'' + \sigma X' + \sigma X = 0$$

$$T' + \sigma T = 0$$

7. [10 points] Let $f(x)$ be an even periodic function with period $2L$. (Thus, the b_n coefficients of its Fourier series are all zero.) If the function enjoys the additional symmetry $f(x) = -f(L-x)$ it can be shown that for even values of n, $a_n = 0$.

   a. Prove that $a_0 = 0$. Hints: Break up the integral $\frac{2}{L}\int_0^L f(x)\,dx$ at $L/2$. The substitution $u = L - x$ may be helpful at a certain point.

   b. [Bonus Problem: 10 points] Prove the general case, $a_n = 0$ for n even.