Review of Even and Odd Functions

| Even: $f(-x) = f(x)$ | Odd: $f(-x) = -f(x)$. |

Even Examples: $5x^6 - 3x^4 + 2$, $\frac{x^4+1}{x^2-7}$, $|x|$, $\cos(x)$, $\cos(x^5)$, $\sin^4(x)$.

Odd Examples: $x^{1/3}$, $x^7 - 6x^3$, $x|x|$, $\sin(x)$, $\sin^5(x^7)$.

Neither: $x^2 + x$, $x + \cos(x)$, $\frac{1}{x+1}$.

Problem: Let $f(x)$ be an odd function and suppose it is definied at $x = 0$. What is $f(0)$?

Problem: Find a function with domain the whole real line that is both even and odd. (There is only one correct answer.)

Fact: If $f(x)$ is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

Fact: If $f(x)$ is odd, then $\int_{-a}^{a} f(x)\,dx = 0$. Draw pictures to see intuitively why these two facts hold.

Fact: If $f(x)$ is even and differentiable, then $f'(x)$ is odd, and vice versa. This can be proved with the Chain Rule. Suppose $f(x)$ is even and differentiable. Then $(f(-x))' = f'(-x)(-1)$. But $f(-x) = f(x)$, so $(f(-x))' = (f(x))' = f'(x)$. Thus, $f'(-x)(-1) = f'(x)$, or $f'(-x) = -f'(x)$. You can also draw pictures of tangent lines to curves to see this intuitively.

Exercises: Suppose that $f(x)$ and $g(x)$ are even and the $h(x)$ and $k(x)$ are odd. Then show that:

(a) $f(x) + g(x)$ is even.

(b) $f(x)g(x)$ is even.

(c) $f(x)h(x)$ is odd.

(d) $h(x)k(x)$ is even.

(e) $h(x) + k(x)$ is odd.

(g) $f(g(x))$ is even.

(h) $f(g(x))$ is even.

(i) $f(h(x))$ is even.

(j) $h(f(x))$ is odd.

(k) $h(k(x))$ is odd.

(f) $f(x) + h(x)$ need not be even or odd.

Example: Let $p(x) = f(x)h(x)$. Then $p(-x) = f(-x)h(-x) = f(x)(-h(x)) = -f(x)h(x) = -p(x)$. Hence we have an odd function.

| You may be tested on this! |