2.6 #4 There are two ways to do this one. Both are correct even though the answers look very different at first. Here is what most of you did.

$$\underbrace{(2xy^2+2y)}_{M} + \underbrace{(2x^2y+2x)}_{N}\,y' = 0$$

$\frac{\partial M}{\partial y} = 4xy+2$ $\quad \frac{\partial N}{\partial x} = 4xy+2$. Hence exact.

Let $\psi = \int M\,dx = x^2y^2+2xy+G(y)$

But also $\psi = \int N\,dy = x^2y^2+2xy+C_2(x)$.

Thus $\psi(x,y) = x^2y^2+2xy$ will work.

General solution is

$\boxed{x^2y^2+2xy = C}$ This is the answer the back of the book had.

Here is how the solutions Manual the grader uses did it. We can divide through by 2 to get $(xy^2+y)+(x^2y+x)y'=0$. But we can simplify even more: $(xy+1)y + (xy+1)xy' = 0$.

Divide through by $(xy+1)$ to get $y + xy' = 0$.

Now $M = y$ and $M_y = 1$, $N = x$ and $N_x = 1$. Exact.

$$\psi = \int M\,dx = xy + C_1(y)$$

$$\psi = \int N\,dy = xy + C_2(x).$$

Let $\psi(x,y) = xy$.

General solution is $\boxed{xy = C.}$

Note: $y + xy' = 0$ is also separable.

But how can these two both be correct?

Watch: Suppose $xy = c$.

Then $x^2y^2 + 2xy = c^2 + 2c$ a constant!

Suppose $x^2y^2 + 2xy = c$.

Then $x^2y^2 + 2xy - c = 0$

and $$xy = \frac{-2 \pm \sqrt{4+4c}}{2} = -1 \pm \sqrt{1+c}.$$

Effectively a constant!

Thus the level curves of $xy = c$ and $x^2y^2 + 2xy = c$ will be the same collection of curves as C varies.

If the grader took off a point for using Method I, let me know and I'll restore it.