Hwk Set 2 Solutions

I. 1 $y = -6e^{x}$

2 $y = Ce^{x}$ $y(1) = Ce^{1} = e^{2} \rightarrow C = e$ $\boxed{y = e\,e^{x} = e^{x+1}}$

3 $y = \frac{1}{2}\sin(2x) + 2x + C$ $y(\pi) = 8$

$y(\pi) = 2\pi + C$, $C = 8 - 2\pi$

$$\boxed{y = \tfrac{1}{2}\sin(2x) + 2x + (8 - 2\pi)}$$

4. $\int y^{-2}\,dy = \int e^{x}\,dx$

$-\frac{1}{y} = e^{x} + C$

$y = \frac{-1}{e^{x}+C}$ $y(0) = 2$

$y(0) = \frac{-1}{1+C} = 2 \Rightarrow C = -3/2$

$$\boxed{y = \frac{-1}{e^{x} - 3/2} = \frac{1}{3/2 - e^{x}}}$$

5. $\int e^{-y}\,dy = \int e^{x}\,dx$

$-e^{-y} = e^{x} + C$

$e^{-y} = C - e^{x}$

$e^{y} = \frac{1}{C - e^{x}}$

$\boxed{y = \ln\left(\frac{1}{C - e^{x}}\right)}$ general solution

6 $\int \sec^2 y \, dy = \int x^2 \, dx$

$\tan y = \frac{1}{3}x^3 + C$

$\boxed{y = \arctan\left(\frac{x^3}{3} + C\right)}$ is gen. sol.

7. Same as 4. $\boxed{y = \frac{-1}{e^x + C}}$ gen. sol.

8. $\int \tan y \, dy = \int x \sin x \, dx$

$\ln|\cos y| = \sin x - x\cos x + C$

$\cos y = \pm e^{\sin x - x\cos x + C} = \underbrace{\pm e^{C}}_{"C"} e^{\sin x - x\cos x}$

$y = \cos^{-1}\left(C e^{\sin x - x\cos x}\right)$ gen sol.

$y\left(\frac{\pi}{4}\right) = 6.$

$\cos^{-1}\left(C e^{\frac{1}{\sqrt{2}} - \frac{\pi}{4}\frac{1}{\sqrt{2}}}\right) = 6$

$C = \cos(6)\, e^{-\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}} \approx 0.96017\ldots$

9. $y' = y^2x^3 + y^2x + 2y^2 + x^3 + x + 2$

$\frac{dy}{dx} = y^2(x^3+x+2) + (x^3+x+2)$

$\frac{dy}{dx} = (y^2+1)\cdot(x^3+x+2)$

$\int \frac{1}{y^2+1}\,dy = \int x^3+x+2\,dx$

$\arctan(y) = \frac{1}{4}x^4 + \frac{1}{2}x^2 + 2x + C$

$y = \tan\left(\frac{1}{4}x^4 + \frac{1}{2}x^2 + 2x + c\right)$

$y(0) = 1$

$y(0) = \tan(C) = 1 \Rightarrow C = \tan^{-1}(1) = \pi/4.$

$$\boxed{y = \tan\left(\frac{x^4}{4} + \frac{x^2}{2} + 2x + \frac{\pi}{4}\right)}$$

10. $\int y^{-3}\,dy = \int x\sqrt{1+x^2}\,dx$ $\quad u = 1+x^2 \quad du = 2x\,dx$

$\frac{y^{-2}}{-2} = \frac{1}{2}\frac{(1+x^2)^{3/2}}{\frac{3}{2}} + C$

$\frac{1}{y^2} = -\frac{2}{3}(1+x^2)^{3/2} + C$

$\frac{1}{y} = \pm\sqrt{C - \frac{2}{3}(1+x^2)^{3/2}}$

cont.

#10 cont. $y(0) = 1$.

$$1 = y(0) = \frac{1}{\pm\sqrt{C - \frac{2}{3}(1+x^2)^{3/2}}} = \frac{1}{+\sqrt{C - \frac{2}{3}}}$$

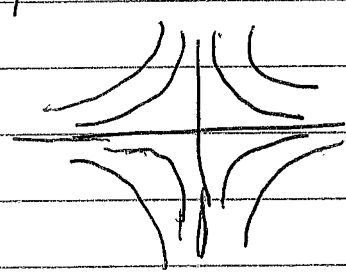
$$\sqrt{C - \tfrac{2}{3}} = 1$$

$$C = \frac{5}{3}$$

$$y(x) = \left(\frac{5 - 2(1+x^2)^{3/2}}{3}\right)^{-\frac{1}{2}}$$

II

1. $y \neq 2$.
2. $2y \neq -x$
3. $x \neq 3$ and $y \neq -4$
4. $y^2 \neq 1$ or $y \neq \pm 1$.
   and $x \neq (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$
5. $\cot\theta$ is not defined at $\theta = n\pi$
   We need $xy \neq n\pi$ $\forall n \in \mathbb{Z}$.
   So, any point not on these hyperbolas is ok.



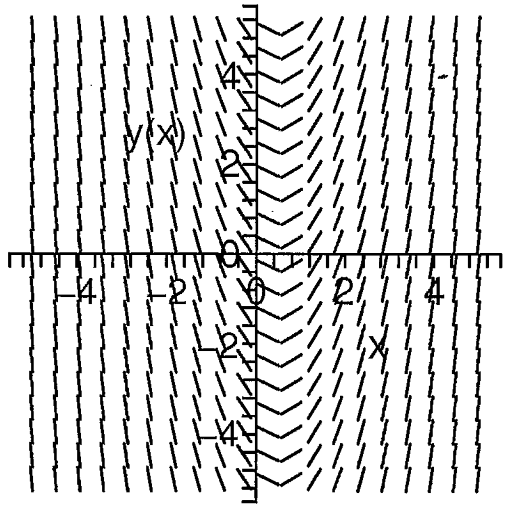
6. $y^3 \neq -x$
7. $\frac{\partial}{\partial y} = \frac{2}{3}x^{\frac{2}{3}}y^{-\frac{1}{3}}$, so $y \neq 0$ is the only restriction.

They need to do these by hand.

```
> with(DEtools):
```

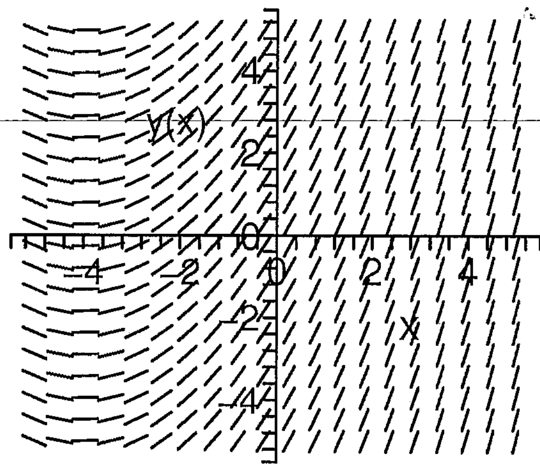
III, #1

```
> dfieldplot(diff(y(x),x)  = 2*x-1, y(x),
  x=-5..5,y=-5..5,arrows=line,color=black,thickness=3);
```



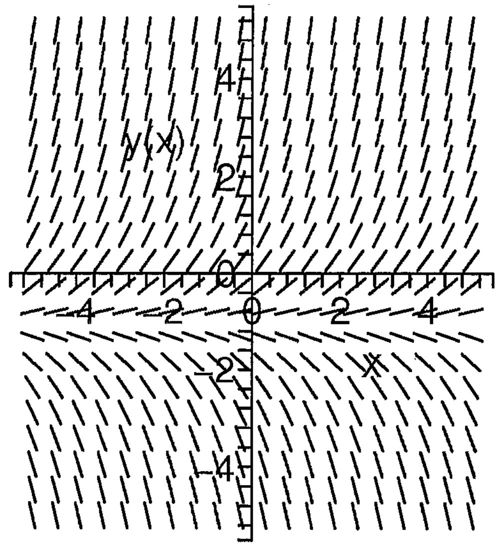
III, #2

```
> dfieldplot(diff(y(x),x)  = x/2 + 2, y(x),
  x=-5..5,y=-5..5,arrows=line,color=black,thickness=3);
```



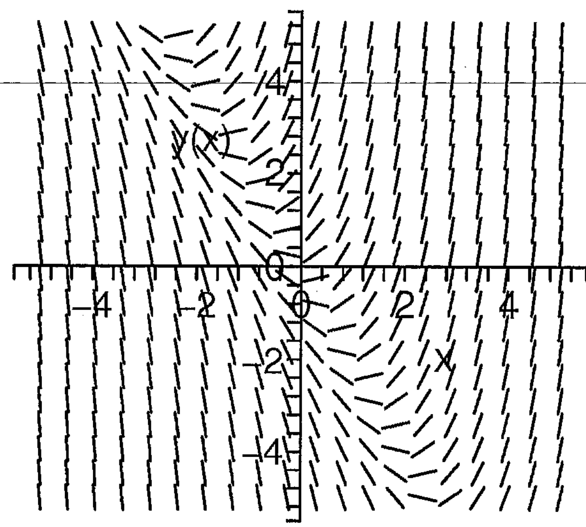
III, #3

```
> dfieldplot(diff(y(x),x)  = y(x)+1, y(x),
  x=-5..5,y=-5..5,arrows=line,color=black,thickness=3);
```



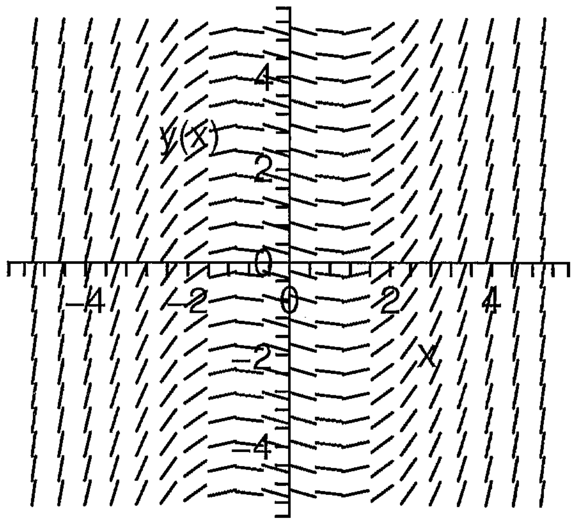
III, #4

```
> dfieldplot(diff(y(x),x)  = 2*x+y(x), y(x),
  x=-5..5,y=-5..5,arrows=line,color=black,thickness=3);
```



III, #5

```
> dfieldplot(diff(y(x),x)  = (x^2-1)/3, y(x),
  x=-5..5,y=-5..5,arrows=line,color=black,thickness=3);
```




```
>
```