

Name: _____ Section time: _____

SCIENTIFIC CALCULATORS ALLOWED

1. [15 points] Find the general solution to $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$. Then find the particular solution that satisfies $y(0) = 3$. What is its domain?

$$\int y \, dy = \int \frac{x^2}{1+x^3} \, dx \quad u = 1+x^3$$

$$\frac{1}{2} y^2 = \frac{1}{3} \ln |1+x^3| + C \quad du = 3x^2 dx$$

$$y = \pm \sqrt{\frac{2}{3} \ln |1+x^3| + C} \quad \text{gen. sol.}$$

$$y(0) = \pm \sqrt{\frac{2}{3} \ln 1 + C} = \pm \sqrt{C} = 3$$

So use + and $C = 9$.

The function $y(x)$ is cont. on $(-1, \infty)$.

2. [10 points] Find the general solution to $\frac{dy}{dx} - 2y = 4x$.

$$u = e^{-2x}$$

$$y' e^{-2x} - 2e^{-2x} y = 4x e^{-2x}$$

$$(y e^{-2x})' = 4x e^{-2x}$$

$$y e^{-2x} = 4 \int x e^{-2x} \, dx = uv - \int v \, du = -\frac{x}{2} e^{-2x} - \int -\frac{1}{2} e^{-2x} \, dx$$

$$\text{Let } u = x$$

$$\text{and } dv = e^{-2x} \, dx.$$

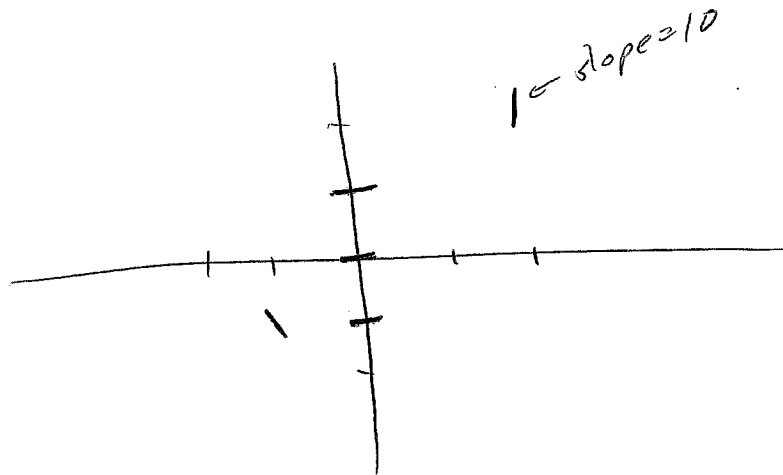
$$\text{Then } du = dx \text{ and}$$

$$v = -\frac{1}{2} e^{-2x}$$

$$= -\frac{x}{2} e^{-2x} + \frac{1}{4} e^{-2x} + C$$

$$= -\frac{1}{4} (2x+1) e^{-2x} + C$$

3. [5 points] Consider $y' = 3x + xy$. Plot the direction field for the point $(0, 0)$, $(0, -1)$, $(2, 2)$, $(-1, 1)$ and $(0, 1)$.



4. [10 points] Consider $y'(t) = F(y)$ where $F(y)$ is given by the graph below. Sketch several solution curves including all equilibrium solutions. Identify the stability type of each equilibrium solution as stable, unstable or semi-stable.

