Math 305 Quiz 1 B Fall 2012

Name: ____________________ Section time: ____________________

SCIENTIFIC CALCULATORS ALLOWED

1. [15 points] Find the general solution to $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$. Then find the particular solution that satisfies $y(0) = 2$. What is its domain?

$\int y\,dy = \int \frac{x^2}{1+x^3}\,dx$ $\quad u = 1+x^3$, $du = 3x^2dx$

$\frac{1}{2}y^2 = \frac{1}{3}\ln|1+x^3| + C$

$y = \pm\sqrt{\frac{2}{3}\ln|1+x^3| + C}$

$y(0) = 2$ so $\pm\sqrt{\frac{2}{3}\ln(1)+C} = 2$

use + $\quad$ " $\sqrt{C} = 2$

$C = 4.$

$y = \sqrt{\frac{2}{3}\ln(1+x^3)+4}$

domain is $x > -1$.

2. [10 points] Find the general solution to $\frac{dy}{dx} - 2y = 4x$.

$\mu = e^{-2x}$

$y'e^{-2x} - 2e^{-2x}y = 4xe^{-2x}$

$(ye^{-2x})' =$

$y = \frac{\int 4xe^{-2x}}{e^{-2x}}$

$= \frac{-2xe^{-2x} - e^{-2x} + C}{e^{-2x}} = -(2x+1) + Ce^{2x}$

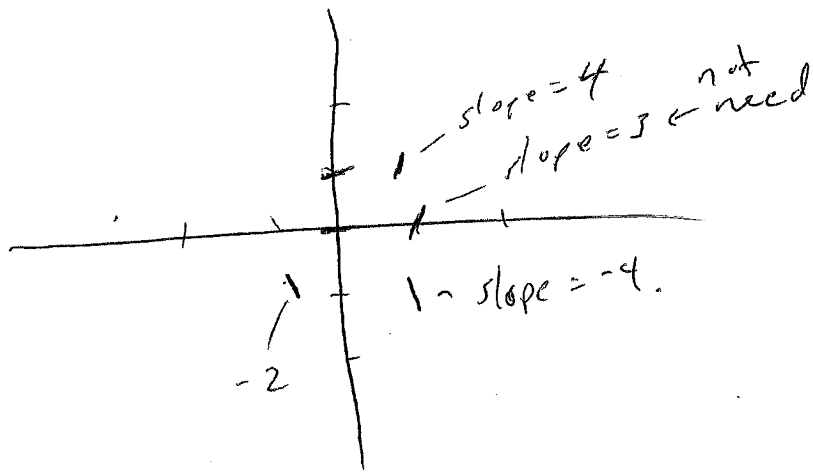
$\int xe^{-2x}dx$

$u = x \quad du = dx$

$dv = e^{-2x}dx \quad v = -\frac{1}{2}e^{-2x}$

$uv - \int v\,du = -\frac{x}{2}e^{-2x} + \int \frac{1}{2}e^{-2x}dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C$

3. [5 points] Consider $y' = 3x + xy$. Plot the direction field for the point $(0, 0)$, $(0, 1)$, $(1, 1)$, $(-1, -1)$ and $(\underset{-1}{\cancel{0}}, 1)$.



4. [10 points] Consider $y'(t) = F(y)$ where $F(y)$ is given by the graph below. Sketch several solution curves including all equalibrium solutions. Identify the stability type of each equalibrium solution as stable, unstable or semi-stable.