Math 305 Quiz 3 Fall 2012

Name: Typo included version

NONGRAPHING CALCULATORS ALLOWED

1. [10 points] Find the solution to the initial value problem below.

$y'' + 2y' = x - 2$

$y'' + 2y' + 2 = x \quad y(0) = 1 \quad y'(0) = 2$

$y'' + 2y' = 0$

$r^2 + 2r = 0$

$r(r+2) = 0$

$r = 0, -2.$

$y_h = c_1 e^{0x} + c_2 e^{-2x}$

$= c_1 + c_2 e^{-2x}.$

$y_p = Ax + B$ won't work b/c B is a solution to the homogeneous problem.

$y_p = Ax^2 + Bx$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A + 4Ax + 2B = x - 2$

$A = \frac{1}{4}$

$\frac{1}{2} + 2B = -2$

$B = \frac{-2.5}{2} = -\frac{5}{4}$

$y = y_h + y_p = c_1 + c_2 e^{-2x} + \frac{1}{4}x^2 - \frac{5}{4}x$

$y(0) = c_1 + c_2 = 1$

$y'(x) = -2c_1 e^{-2x} + \frac{1}{2}x - \frac{5}{4}$

$y'(0) = -2c_1 - \frac{5}{4} = 2$

$-8c_1 - 5 = 8 \quad c_1 = -\frac{13}{8} \qquad c_2 = 1 + \frac{13}{8} = \frac{21}{8}$

2. [5 points. each] For each pair of functions determine if it is linearly independent or dependent on the given interval.

a. $\{\ln x^2, \ln x^3\}$ on $(0, \infty)$.

b. $\{\sin x^2, \cos x^2\}$ on the real line.

c. $\{1 + \cot^2 x, 4\csc^2 x\}$ on $(0, \pi)$.

(a) $\ln x^2 = \frac{2}{3}\ln x^3$ since

$2\ln x = \frac{2}{3} \cdot 3 \ln x.$

L.D.

b. $W = -2x$ is not always zero.

Thus L.I.

c. $1 + \cot^2 x = \csc^2 x$

Thus L.D.

Math 305 Quiz 3 Fall 2012

Name: Typo fixed version

NONGRAPHING CALCULATORS ALLOWED

1. [10 points] Find the solution to the initial value problem below.

$$y'' + 2y' + 2y = x \quad y(0) = 1 \quad y'(0) = 2$$

$r^2 + 2r + 2 = 0$

$r = \frac{-2 \pm \sqrt{4-8}}{2}$

$= -1 \pm i$

$y_h = C_1 e^{-t}\sin t + C_2 e^{-t}\cos t.$

Let $y_p = Ax + B.$

$y_p' = A$

$y_p'' = 0$

plug in:

$2A + 2(Ax + B) = x$

$2Ax + (2A + 2B) = x$

$A = \frac{1}{2} \quad B = -1.$

$y_p = \frac{1}{2}x - 1$

$y = y_h + y_p = C_1 e^{-t}\sin t + C_2 e^{-t}\cos t + \frac{1}{2}x - 1$

$y(0) = C_2 - 1 = 1 \Rightarrow C_2 = 2.$

$y'(t) = -C_1 e^{-t}\sin t + C_1 e^{-t}\cos t - C_2 e^{-t}\cos t - C_2 e^{-t}\sin t + \frac{1}{2}$

$y'(0) = 0 \quad C_1 - C_2 - 0 + \frac{1}{2} = 2$

$C_1 = C_2 - \frac{1}{2} + 2 = 2 - \frac{1}{2} + 2 = 3\frac{1}{2}$ or $\frac{7}{2}$

2. [5 points each] For each pair of functions determine if it is linearly independent or dependent on the given interval.

a. $\{\ln x^2, \ln x^3\}$ on $(0, \infty)$. $\longrightarrow$ $\ln x^2 = \frac{2}{3}\ln x^3$ since $2\ln x = \frac{2}{3}\cdot 3\ln x$. Thus L. D.

b. $\{\sin x^2, \cos x^2\}$ on the real line.

c. $\{1 + \cot^2 x, 4\csc^2 x\}$ on $(0, \pi)$.

$W = -2x$ which is not always zero. Thus L. I.

c. $1 + \cos^2 x = \frac{1}{4}(4\csc^2 x)$, by trig identity.

Thus L. D.