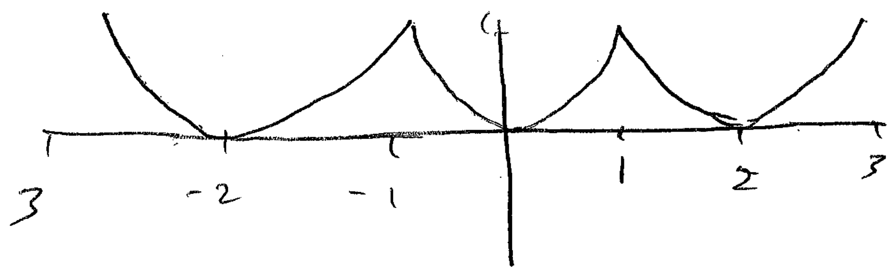
Math 305 Quiz 4 Fall 2012

Name: Key Section time: ____________

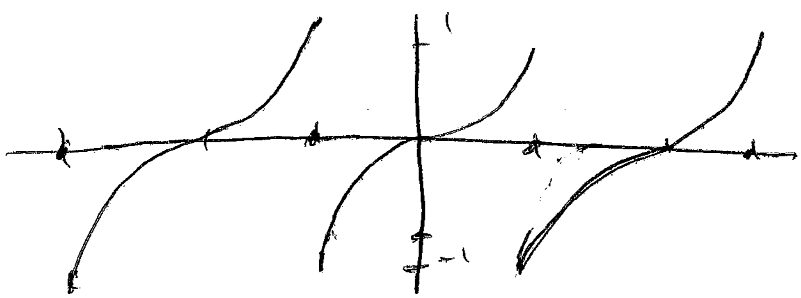
NON-GRAPHING CALCULATORS ALLOWED

1. [10 points] Consider the function $f(x) = x^2$ defined on $[0, 1]$.

   a. Draw its even periodic extension over $[-3, 3]$.



   b. Draw its odd periodic extention over $[-3, 3]$.



2. [10 points] Show that the PDE $u_{xx} = u_{tt} + u_t$ is separable.

Suppose $u = XT$.

Then $X''T = XT'' + XT'$

Thus $\dfrac{X''}{X} = \dfrac{T'' + T'}{T}$,

So the PDE is separable.

5. [15 points] Consider $y'' + xy = 0$. Find the general power series solution centered about $x_0 = 0$. Find a recursive formula for a_n. Show that no matter what the initial conditions are that every third term in the power series solution is zero.

$$y = \sum_{n=0}^{\infty} a_n x^n \qquad xy = \sum_{n=0}^{\infty} a_n x^{n+1} = \sum_{n=1}^{\infty} a_{n-1} x^n$$

$$y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

Notice has an $n=0$ term, but xy doesn't.

$y'' + xy =$

$$2\cdot 1\, a_2 x^0 + \sum_{n=1}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} a_{n-1} x^n = 0$$

$a_2 = 0$ and for $n \geq 1$ $\quad a_{n+2} = \dfrac{-a_{n-1}}{(n+2)(n+1)}$.

Assum a_0 & a_1 are given.

$a_3 = \dfrac{-a_0}{3\cdot 2}$ $\quad n=1$

$a_4 = \dfrac{-a_1}{4\cdot 3}$ $\quad n=2$

$a_5 = \dfrac{-a_2}{5\cdot 4} = 0$!

$a_8 = \dfrac{-a_5}{8\cdot 7} = 0$!

$a_{11} = \dfrac{-a_8}{11\cdot 10} = 0$ $\qquad$ so every third term must be zero!

3. [10 points] Suppose that $g(x)$ is an odd function and $f(x)$ is an even one. For each function below determine if it is even, odd or not necessarily either. justify your answers.

   a. $h(x) = g^2(x)f(g(x))$.

   $h(-x) = [g(-x)]^2 f(g(-x)) = [-g(x)]^2 f(-g(x)) = g(x)^2 f(g(x)) = h(x)$.

   Hence even.

   b. $Q(x) = g(g(x))f(x)$

   $Q(-x) = g(g(-x))f(-x) = g(-g(x))f(x) = -g(g(x))f(x) = -Q(x)$.

   odd

   c. $H(x) = g(f'(x))$

   f even $\Rightarrow f'$ odd. $H(-x) = g(f'(-x)) = g(-f'(x)) = -g(f'(x)) = -H(x)$

   odd

   d. $P(x) = g(x)[f(x) + g(x)]$

   neither. Ex. Let $f(x) = 1$, $g(x) = x$. Then $P(x) = x + x^2$ $P(-1) = 0 \neq$ $P(1) = 2$ or $-P(1) = -2$.

   e. $J(x) = f(x) + xg(x)$.

   $J(-x) = f(-x) + (-x)g(-x) = f(x) + (-x)(-g(x)) = f(x) + xg(x) = J(x)$.

   Even

4. [10 points] Consider $y'' + 3y' + e^x y = 0$ with $y(0) = 1$ and $y'(0) = 2$. Find the first five terms of the power series solution centered about $x_0 = 0$.

$a_0 = 1$ $a_1 = 2$. $y'' = -3y' - e^x y$.

$y''(0) = -3y'(0) - e^0 y(0) = -3 \cdot 2 - 1 \cdot 1 = -7$.

$a_2 = \frac{-7}{2!} = \frac{-7}{2}$

$y''' = -3y'' - e^x y - e^x y'$

$y'''(0) = -3 \cdot (-7) - 1 \cdot 1 - 1 \cdot 2 = 21 - 3 = 18$

$a_3 = \frac{18}{3!} = 3$

$y'''' = -3y''' - e^x y - 2e^x y' - e^x y''$

$y''''(0) = -3(18) - 1 \cdot 1 - 2 \cdot 1 \cdot 2 - 1(-7)$

$-54 - 1 - 4 + 7 = -52$

$a_4 = \frac{-52}{4!} = \frac{-52}{120} = \frac{-13}{30}$