Math 305 Homework Set 8: Based on Chapter 5.1-5.3 Fall 2012

Due Friday October 26

5.1. 13, 16, 21, 24, 28. (These are just review problems on power series.)

"5.2". These problems are based on the methods of 5.2.

1. Find the general series solution to each different equation below including a recursive formula for the a_n's.

a. $y'' + xy' - y = 0$ centered at $x_0 = 0$.

b. $xy'' + xy' + 3y = 0$ centered at $x_0 = 2$.

c. $y'' + x^3y' + 3y = 0$ centered at $x_0 = 0$.

2. For solution for problem 1.a do the following. Let y_1 be the solution obtained from the initial conditions $y(0) = 1$ and $y'(0) = 0$; Let y_2 be the solution obtained from the initial conditions $y(0) = 0$ and $y'(0) = 1$.

a. Explain why we know they must be linearly independent.

b. Write out the first six terms of $y_1(x)$ and all the terms of $y_2(x)$!

c. Use a computer to find the first 10 terms of the series soltion when $y(0) = 2$ and $y'(0) = 1$. Here is the command used in Maple.

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> with(DEtools);
> Oreder:= 10;
> dsolve({diff(y(x),x$2)+x*diff(y(x),x)-y(x)=0,y(0)=2, D(y)(0)=1},y(x),series);
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d. Let $y_{1,k}$ be the first k terms of the series you found in (c). On a computer plot $y_{1,3}$, $y_{1,5}$, $y_{1.7}$ and $y_{1,9}$ for $x \in [0, 5]$.

"5.3". These problems are based on the methods of 5.3.

1. For each initial value problem find the first five terms of the series solution ("zero" terms count).

a. $y'' - (\sin x)y' - 3x^2y = 0$, $y(0) = 1$, $y'(0) = 2$.

b. $e^x y'' - 2y' + xy = 0$, $y(0) = 2$, $y'(0) = 0$.

c. $y'' - x^2y' - 3y$, $y(3) = 1$, $y'(3) = 1$.

2. Find a lower bound for the radius of convergence of the series solution to each of the following differential equations.

a. $(1 + x^3)y'' + 4xy' + y = 0$ centered at $x_0 = 3$.

b. $y'' + \dfrac{1}{x-2}y' + \dfrac{1}{x^2+3}y = 0$ centered at $x_0 = 1$.