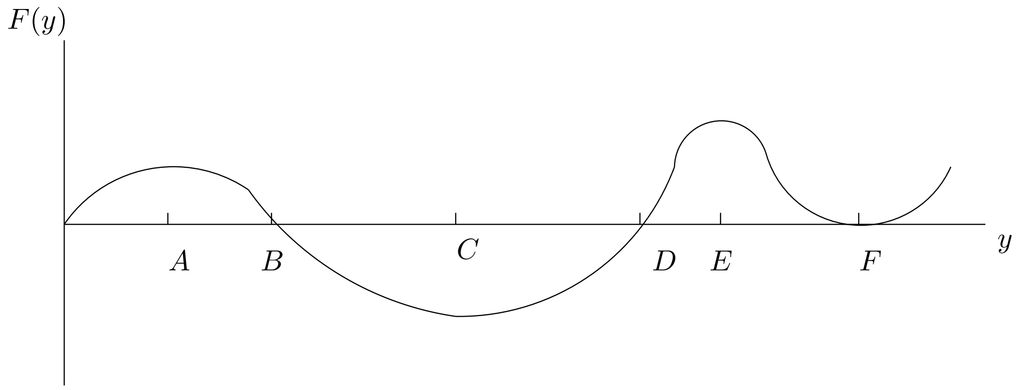
Math 305 Homework Set 12 Spring 2017

Due Friday, April 21

I. Solve the initial value problems.

1. $y'' + 4y = 3\sin 2t$, $y(0) = 2$, $y'(0) = 0$.
2. $y'' - 2y' + y = te^t + 4$, $y(0) = 1$, $y'(0) = 0$.

II. Suppose $y'(t) = F(y(t))$, where the graph of $F(y)$ is given below. Carefully draw several solution curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are non-negative.



III. These are similar to problems 1-6 in 10.5.

1. Consider the PDE, $p(x)u_{xx} + u_t = 0$. Suppose $u(x,t) = X(x)T(t)$. Show that for some constant σ it follows that

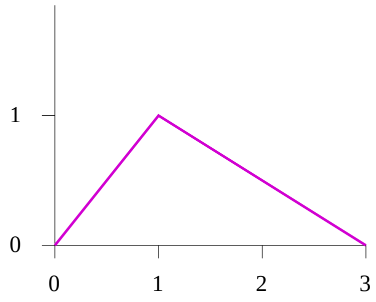
$$p(x)X'' + \sigma X = 0 \qquad T' - \sigma T = 0.$$

2. Consider the PDE, $u_{xx} + u_{yy} = u_t$. Suppose $u(x,y,t) = X(x)Y(y)T(t)$. Show that for some constants α and σ it follows that

$$T' + \sigma T = 0 \qquad X'' + \alpha X = 0 \qquad Y'' + (\sigma + \alpha)Y = 0.$$

IV. Now for the fun stuff.

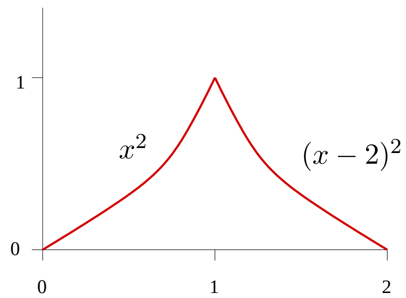
1. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = 3$, with initial displacement $f(x)$ given below.



Find $u(x, t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

2. Consider a vibrating ideal string (that is assume the wave equation is valid) with $a = 1$, $L = \pi$, with initial displacement $f(x) = \sin 5x$. Find $u(x, t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

3. Consider a metal rod of length 2 and $\alpha = 1$ with the initial temperature shown below.



Find $u(x, t)$ and plot it for $t = 0.0$, $t = 0.001$, $t = 0.01$, $t = 0.1$, and $t = 0.5$.

4. Animate any of these for extra credit. You can just show me the animation on your laptop/tablet or email it to me.