Due Monday February 27

I. For each differential equation below, find the general solution.

(1) $9y'' + 6y' + y = x^2$
(2) $9y'' + 6y' + y = e^{-\frac{x}{3}}$
(3) $y'' + 3y' - 4y = 2x + e^x$
(4) $y'' + 3y' - 4y = \sin x$
(5) $y'' + 4y = x$
(6) $y'' + 4y = \cos x$
(7) $y'' + 4y = \cos 2x$

II. Determine if each pair of functions is linearly dependent or independent on the given interval.

(1) $\{x^2 + x, x\}$ on $(-\infty, \infty)$
(2) $\{\sin x, \sin 2x\}$ on $(-\infty, \infty)$
(3) $\{\ln x, \ln \frac{1}{x}\}$ on $(0, \infty)$
(4) $\{e^x, e^{x+7}\}$ on $(-\infty, \infty)$
(5) $\{\frac{x+1}{x^2-1}, \frac{x}{x^2-1}\}$ on $(-1, 1)$

III. For a set of three functions, $\{f_1, f_2, f_3\}$, each twice differentiable, the Wronskian is defined to be

$$\det \begin{bmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{bmatrix}$$

A set of three functions defined on an interval I is linearly dependent if there exist real numbers, C_1, C_2 and C_3, not all zero such that

$$C_1 f_1(x) + C_2 f_2(x) + C_3 f_3(x) = 0 \qquad (*)$$

for all $x \in I$. Otherwise a set is linearly independent. It is known that if a set of three twice differentiable functions is linearly dependent then the Wronskian is zero on I. (**Extra Credit: Prove this!**)

Find the Wronskian of the four sets below. If it is not always zero conclude the set is linearly independent. If it is always zero find values for C_1, C_2 and C_3, not all zero, that satisfy $(*)$. The interval for each is the whole real line.

(1) $\{\sin^2 x, \cos^2 x, \cos 2x\}$
(2) $\{\sin x, \sin 2x, \sin 3x\}$
(3) $\{x^2 + 3x, x + 2, x^2 + x - 4\}$
(4) $\{x^2, x^2 + x, x^2 + x + 1\}$

IV. We consider differential equations of the form $y'' + p(t)y' + q(t)y = 0$. Assume $p(t)$ and $q(t)$ are continuous on the whole real line.

1. Explain why both $y_1 = t^3$ and $y_2 = t^4$ cannot be solutions.
2. Explain why both $y_1 = \sin t$ and $y_2 = t^2 + t$ cannot be solutions.

V. 1. Find a differential equation of the given form $y'' + p(x)y' + q(x)y = 0$ that has $\{x + 1, x + 2\}$ as a fundamental solution set. On what interval is it valid?

2. Find a differential equation of the given form $y'' + p(x)y' + q(x)y = 0$ that has $\{\sin x, e^x\}$ as a fundamental solution set. On what interval is it valid?