1. [20 points] Consider $y'' + 2 \sin(x)y' - 3y = 0; y(0) = 0, y'(0) = 2$. Find the first four terms of the power series of the solution.
2. [20 points] Consider $y'' + 2xy' - 3y = 0$. Find the general series solution; in particular find a recursive formula for the $a_n$'s.
3. [20 points] Consider \((x^2 + 4)y'' + (x^3 + 1)y' - 4x^2y = 0\). DO NOT TRY TO SOLVE THIS. Merely find a lower bond on the radius of convergence of the series solution centered about \(c = 1\).
4. [20 points] Let $f(x)$ be a periodic function defined by the graph below.

a. Find $a_0$.

b. Find $b_5$. 
5. [20 points] Consider the partial differential equation

\[ U_{xx} - U_{xt} - U_t = 0. \]

Suppose that there is a solution of the form \( U(x,t) = X(x)T(t) \). Show that \( X(x) \) and \( T(t) \) must satisfy the ordinary differential equations below:

\[ X'' + \sigma X' + \sigma X = 0 \]
\[ T' + \sigma T = 0 \]
6. [20 bonus points] Let \( f(x) \) be an even periodic function with period \( 2L \). (Thus, the \( b_n \) coefficients of its Fourier series are all zero.) If the function enjoys the additional symmetry \( f(x) = -f(L - x) \) it can be shown that for even values of \( n \), \( a_n = 0 \).

   a. Prove that \( a_0 = 0 \). Hints: Break up the integral \( \frac{2}{L} \int_0^L f(x) \, dx \) at \( L/2 \). The substitution \( u = L - x \) may be helpful at a certain point.

   b. Prove the general case, \( a_n = 0 \) for \( n \) even.