Print Name: _____ ID number: _____

Fall 2004 **Math 305** **Test 1**

Only Scientific Calculators Allowed

1. [10 points] You are giving presentation at your new job. In the midst of it someone points out that your direction field slides do not match your equations. You realize that your secretary has mixed the slides up! Quickly figure out which direction field matches each equation. (Your secretary also included an extra one you do not need.) Each correct match is worth 2 points, each incorrect match is -1 point.

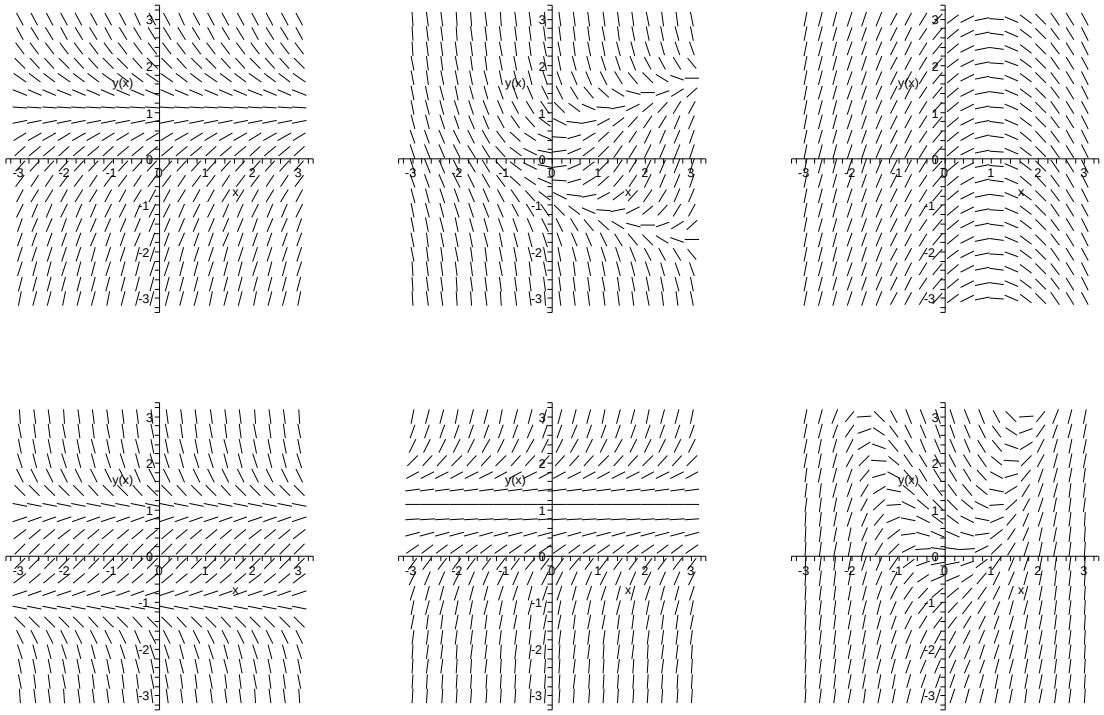
   (a) $y' = x - y^2$

   (b) $y' = 1 - x$

   (c) $y' = 1 - y^2$

   (d) $y' = 1 - y$

   (e) $y' = (1 - y)^2$



2. [15 points] A tank initially contains 100 gallons of fresh water. Salt water with salt concentration of 1 pound per gallon flows in at 1 gallon per minute. The mixture flows out at a rate of 1 gallon per minute. But, water is evaporating at a rate of .25 gallons per minute. How many pounds of salt are in the tank when the water volume reaches 50 gallons? Hint: If you set it up correctly, both the integrals needed are easy.

3. [15 points] Solve $(3x^2y + 2y + 2) + (x^3 + 2x)y' = 0$, with $y(1) = 2$. Find y explicitly as a function of x.

4. [15 points] Find the general solution to $y' = \dfrac{x^2y + xy^2}{x^3}$. Find y explicitly as a function of x.

5. [15 points] Solve $y'y'' = 2$ for $y(0) = 1$ and $y'(0) = 2$. Find y as a function of t. Hint: start by letting $v = y'$.

6. [10 points] Let $y' = f(x, y)$. Recall that a unique solution is guaranteed to exist near those initial conditions, $y(x_0) = y_0$, where $f(x_0, y_0)$ and $\partial_y f(x_0, y_0)$ exist and are continuous. For the two equation below describe the set of points where these conditions are satisfied. Sketch these points in the xy-plane.

   (a) $y' = (x^2 + y^2 - 4)^{1/3}$

   (b) $y' = \dfrac{x^2 + 4}{(x + 3)(y - 2)}$

7. [10 points] Consider the general Bernoulli equation:

$$\frac{dy}{dt} + p(t)y = g(t)y^n \qquad\qquad n \geq 2.$$

   Let $v = y^{1-n}$. Show that this substitution will transform the Bernoulli equation into the linear equation:

$$\frac{dv}{dt} + (1 - n)p(t)v = (1 - n)g(t).$$

   Here are the steps you will need:
   STEP 1: Find y as a function of v.
   STEP 2: Find y^n as a function of v.
   STEP 3: Find $\frac{dy}{dt}$ in terms of v and $\frac{dv}{dt}$ by using the Chain Rule.
   STEP 4: Plug these back into the original equation and simplify.
   Warning: There will be many fractional exponents; keep careful track of them.

8. [10 points] Let $y' = 2y - t$, with $y(0) = 2$. Use Euler's Method to approximate $y(0.2)$ with a step size of $h = 0.1$.

9. [10 Bonus Points] Derive the formula for the Improved Euler's Method. Draw pictures as needed, but describe your reasoning in clear correct English sentences.