Math 305 Quiz 2 Spring 2017

Name: Key

NONGRAPHING CALCULATORS ALLOWED

1. [5 points] Find the general solution to $y' + t^2 y = t^2 y^4$. Let $v = y^{1-n} = y^{1-4} = y^{-3}$.

Then $y = v^{-1/3}$, $y^4 = v^{-4/3}$ and $y' = -\frac{1}{3} v^{-\frac{4}{3}} v'$. Plug in.

$-\frac{1}{3} v^{-\frac{4}{3}} v' + t^2 v^{-\frac{1}{3}} = t^2 v^{-\frac{4}{3}}$

$v' - 3t^2 v = -3t^2$ Linear!

$\mu = e^{-3\int t^2 dt} = e^{-t^3}$

$(e^{-t^3} v)' = -3t^2 e^{-t^3}$

$e^{-t^3} v = \int \_\ dt = \int e^u du = e^{-t^3} + C$

$v = 1 + Ce^{t^3}$.

$y = (v)^{-\frac{1}{3}} = (1 + Ce^{t^3})^{-1/3}$

2. [5+5 points] (a) Find the general solution to $y' = \dfrac{y^2 + 2xy}{x^2}$. Leave as a relation. (b) Solve for y as a funcion of x.

(a)

$y' = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right)$. Let $v = \frac{y}{x}$. Then $y = vx$ so $y' = xv' + v$.

$v + xv' = v^2 + 2v$

$xv' = v^2 + v$

$\int \frac{dv}{v^2+v} = \int \frac{1}{x} dx = \ln x + C$

$\hookrightarrow \int \frac{1}{v(v+1)} dv = \int \frac{1}{v} - \frac{1}{v+1} dx = \ln v - \ln v+1$

$\ln \frac{v}{v+1} = \ln x + C'$

$\frac{y/x}{y/x + 1} = e^{\ln x + C'} = Cx$

(b)

$\frac{y}{y+x} = Cx$

$y = Cxy + x^2$

$y - Cxy = x^2$

$y(1 - Cx) = x^2$

$y = \dfrac{x^2}{1 - Cx}$