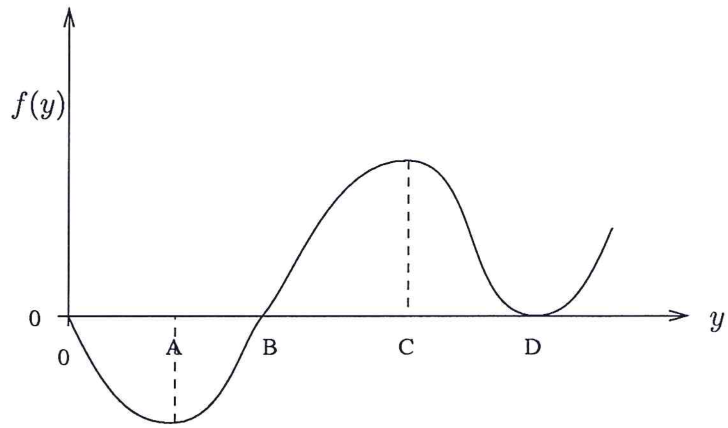
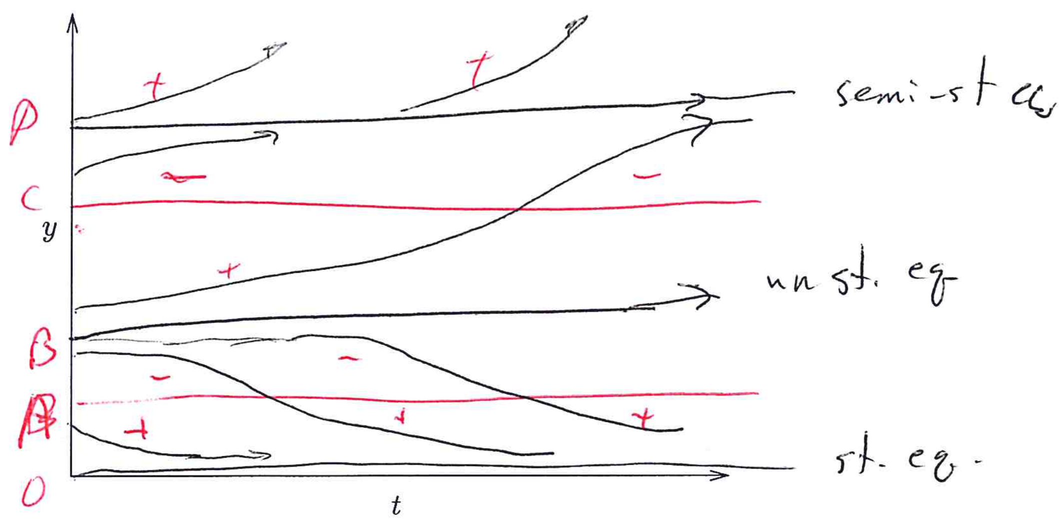
Test 1 **Math 305** **Spring Semester, 1998**

Each problem is worth 20 points.
NO CALCULATORS

1. Suppose $y'(t) = f(y)$, where the graph of $f(y)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are nonnegative.

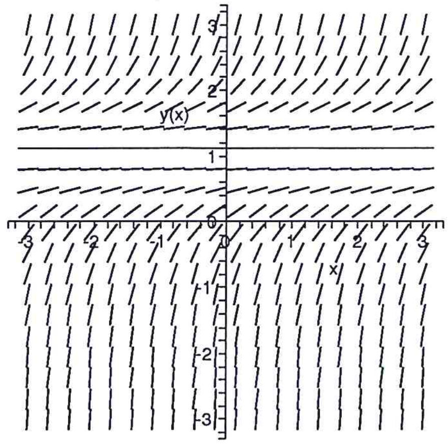




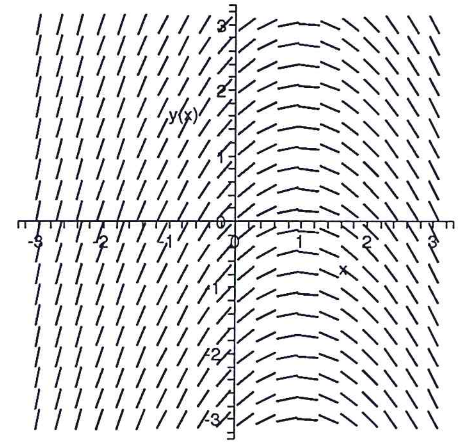
2. Match the differential equation with its direction field. (You get 4 points for each correct match, -2 for each wrong match.)
   1. $y' = y - 2$
   2. $y' = 2 - y$
   3. $y' = |y - 2|$
   4. $y' = y + x$
   5. $y' = x - y$

ignore

$y' = (1-y)^2$

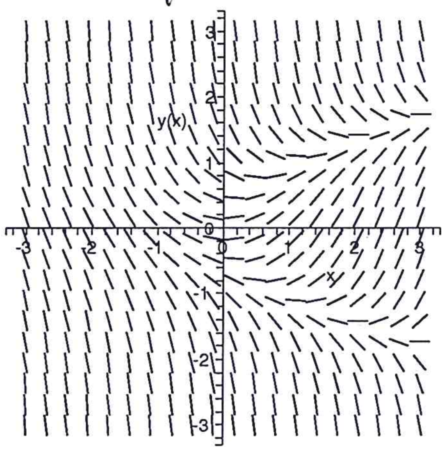


$y' = 1-x$

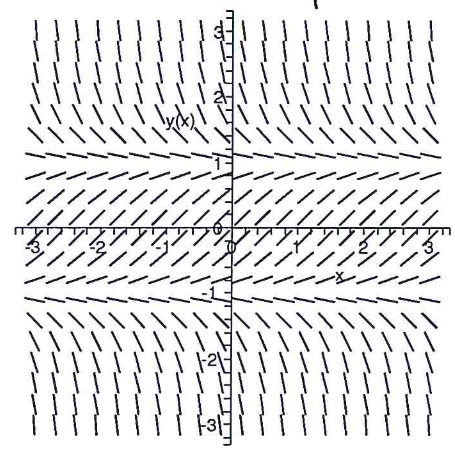


$y' = x-y^2$ ✓

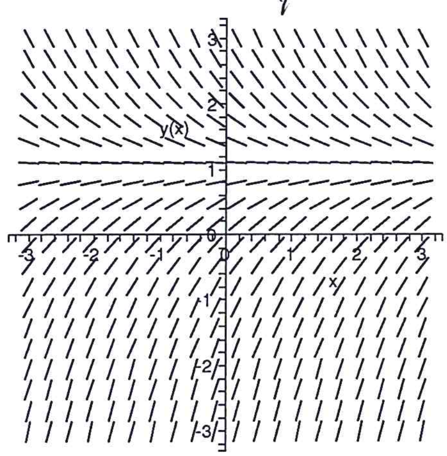
y^2-x
$x-y^2$



$y' = 1-x^2$

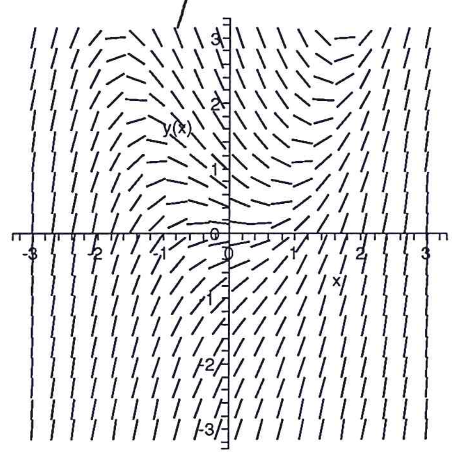


$y' = 1-y$



$y' = x^2-y$

x^2-y
$y-x^2$



#2 The given equations are wrong due to a computer mixup. So I just figured out what the equations had to be.

#3 $0 \le t < 1$ $y' + y = 0$. Thus $y = ce^{-t}$. Since $y(0)=1$, $y = e^{-t}$

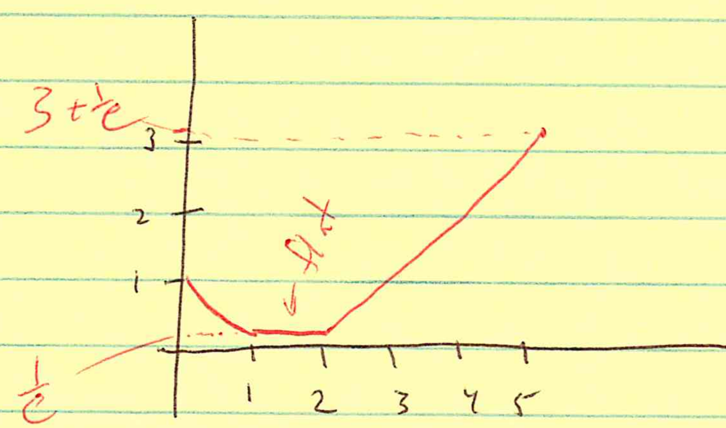
$1 \le t < 2$ $y' = 0$. $y =$ constant. $= \frac{1}{e}$ to make solution cont. at $t=1$.

$t \geq 2$ $y' = 1$, steady linear growth.

$y = t + c$.

$y(2) = 1/e$ so $c = \frac{1}{e} - 2$.

$$y(t) = \begin{cases} e^{-t} & t \in [0,1) \\ e^{-1} & t \in [1,2) \\ t + e^{-1} - 2 & t \in [2,5]. \end{cases}$$



4. $\underbrace{(x^2+y)}_{N}\,y' + \underbrace{2xy-6x}_{M} = 0$

$M_y = 2x \qquad N_x = 2x \qquad$ exact!

$\Psi = \int 2xy - 6x \, dx = x^2y - 3x^2 + C_1(y)$

$\Psi = \int x^2 + y \, dy = x^2y + \frac{1}{2}y^2 + C_2(x)$

Let $\Psi = x^2y - 3x^2 + \frac{1}{2}y^2$

General solution is $\boxed{x^2y - 3x^2 + \frac{1}{2}y^2 = C}$

6.

$Q(0) = .5 \times 200 = 100$ lb

$Q' = .7\,\frac{Q}{V}5 - \frac{Q}{V}5 = -0.3 \times 5\,\frac{Q}{V} \qquad V = 200 - t$

$Q' = \frac{-1.5\,Q}{200-t} \qquad \ln Q = \int \frac{-1.5}{200-t}\,dt$

$= +1.5 \ln(200-t) + C$

$Q = C(200-t)^{1.5} \qquad C = \frac{100}{(200)^{1.5}}$

$\boxed{Q(200) = 0.}$