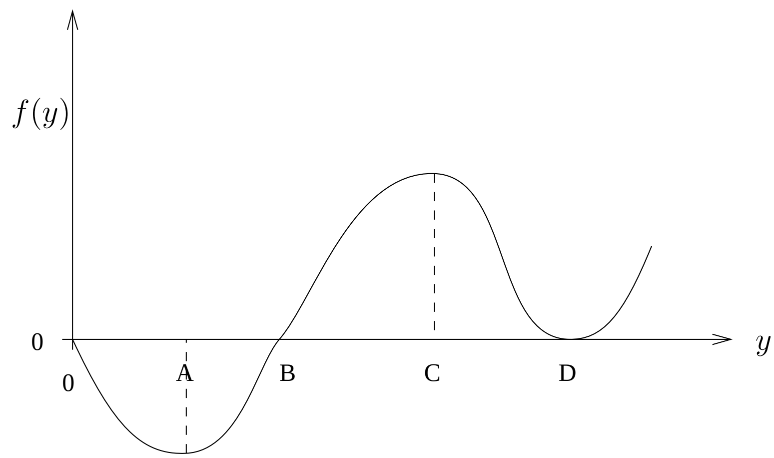
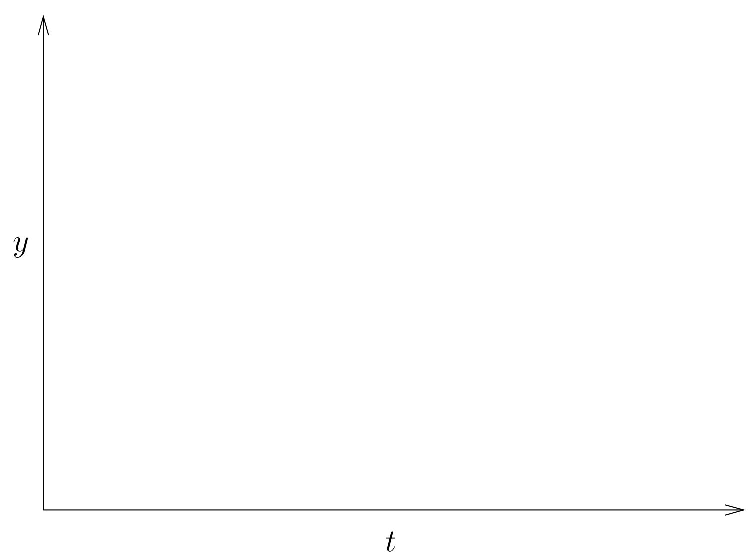
Test 1 **Math 305** **Spring Semester, 1998**

Each problem is worth 20 points.
NO CALCULATORS

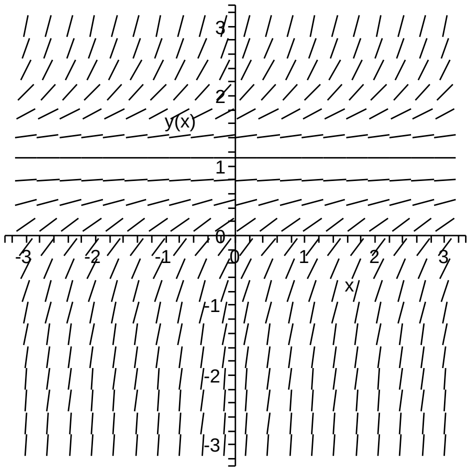
1. Suppose $y'(t) = f(y)$, where the graph of $f(y)$ is given below. Carefully draw the integral curves for this equation. What are the equilibrium solutions? What are their stability types? Describe the initial concavity of the solution curves. Assume $y(t)$ and t are nonnegative.

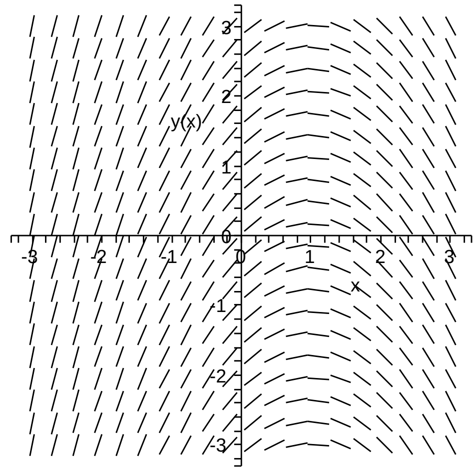


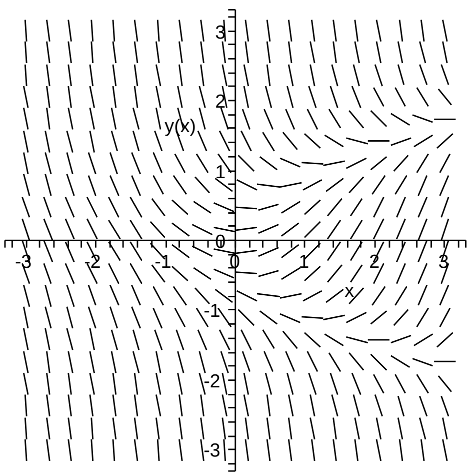


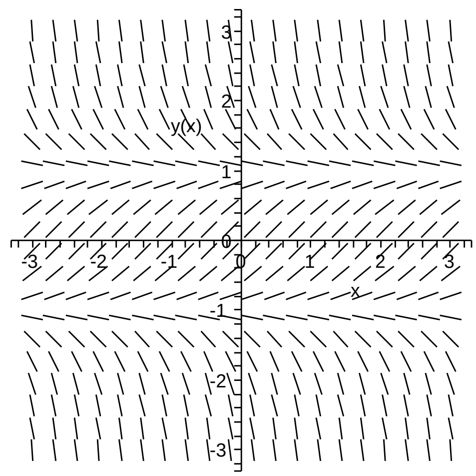
2. Match the differential equation with its direction field. (You get 4 points for each correct match, -2 for each wrong match.)

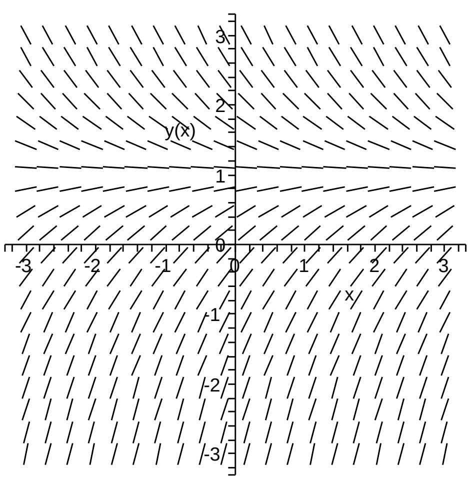
   1. $y' = y - 2$
   2. $y' = 2 - y$
   3. $y' = |y - 2|$
   4. $y' = y + x$
   5. $y' = x - y$

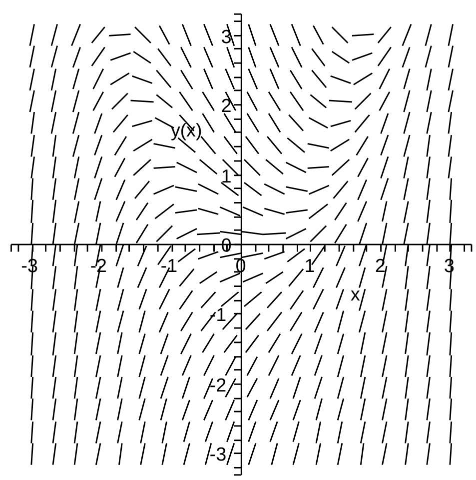

y(x)
x


y(x)
x


y(x)
x


y(x)
x


y(x)
x


y(x)
x

3. Find a continuous solution to the initial value problem, $y' + p(t)y = g(t)$, $y(0) = 1$, where

$$p(t) = \begin{cases} 1 & t \le 1 \\ 0 & t > 1 \end{cases}$$

and

$$g(t) = \begin{cases} 0 & t \le 2 \\ 1 & t > 2 \end{cases}$$

Sketch the graph of your solution, $y(t)$, for $0 \le t \le 5$. Hint: $y(5) = 3 + 1/e$.

4. Find the general solution of the differential equation $(x^2 + y)y' + 2xy = 6x$. Hint: check for exactness. (You need not solve for y.)

5. A tank contains 200 gallons of salt water. The initial salinity is .5 lb/gal. The water is then pumped out of the tank, run through a filter, and pumped back in. The flow rate is 5 gallons a minute. The filter removes 30% of the salt. Also, water is evaporating at a rate of 1 gallon per minute. How much salt is left in the tank when the water has all evaporated? Assume the water in the tank is well mixed.