1. Find the first three nonzero terms of Taylor series of \( \tan x \) centered about \( x = 0 \).

2. Let \( y'' + (x - 1)y' + 3y = 0 \). Use the series method, centered about \( x_0 = 1 \), to find the general solution. You must find a recursive formula for \( a_n \).

3. Let \( 2y'' + y' + \sin(x)y = 0 \). Let \( y = \sum_{n=0}^{\infty} a_n x^n \) be the solution. Let \( y(0) = 1 \) and \( y'(0) = 2 \). Find \( a_2 \), \( a_3 \) and \( a_4 \).

4. Let \( f(x) \) be even and \( g(x) \) be odd. Show that

\[
\int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx
\]

and

\[
\int_{-a}^{a} g(x) \, dx = 0.
\]

5. Find the solution of the heat conduction problem

\[
100u_{xx} = u_t, \quad 0 < x < 1, \quad t > 0
\]

\[
u(0, t) = 0, \quad u(1, t) = 0 \quad t > 0
\]

\[
u(x, 0) = \sin(2\pi x) - 2\sin(5\pi x), \quad 0 \leq x \leq 1.
\]

6. Let \( f(x) \) be a periodic function defined by the graph below. Find its Fourier series.

![Graph with periodic function]

7. The two dimensional heat equation is

\[
\alpha^2 (U_{xx} + U_{yy}) = U_t.
\]

Suppose \( U(x, y, t) = X(x)Y(y)T(t) \). Show that \( X \), \( Y \) and \( T \) must satisfy the ordinary differential equations below:

\[
T' + \sigma_1 \alpha^2 T = 0
\]

\[
X'' + \sigma_2 X = 0,
\]

\[
Y'' + (\sigma_1 + \sigma_2) Y = 0
\]